THE GEOMETRICAL KNOWLEDGE OF MEDIAEVAL MASTER MASONS

BY LON R. SHELBY

DURING the past one hundred and fifty years numerous scholars have searched for the geometrical canons which supposedly were used by master masons in the design and construction of mediaeval churches. But in this search for one of the keys to an understanding of mediaeval architecture, these scholars have seldom asked themselves what was the actual character and content of the geometrical knowledge which a mediaeval master mason might have been expected to possess. This paper attempts to answer that question.

The idea that geometry played a fundamental role in the mediaeval masons' craft is by no means the invention of modern scholars; it was a notion commonly held by mediaeval masons themselves. The thirteenth-century French master mason, Villard de Honnecourt, touched on the matter in the "Preface" to his *Sketchbook*: "Villard de Honnecourt greets you and bids all those who work with the devices found in this book to pray for his soul and to remember him. For in this book one will find good advice concerning the proper technique of masonry and the devices of carpentry. You will also find the technique of drawing — the forms — just as the art of geometry requires and teaches it."[1] In another place Villard commented, "Here begins the technique of the forms of drawing, just as the art of geometry teaches them for working more easily. And on other sheets are those of masonry."[2] Although he does not quite say it, the implication seems to be that "on other sheets will be found the technique of the forms (*li force des trais*) of masonry, likewise as taught by the art of geometry."

An even stronger assertion of the essential role of geometry in the masons' craft was made around 1400 by the unknown author — probably a cleric — who compiled a historical introduction to the "Articles and Points of Masonry" which set forth the customs and regulations pertaining to the masons' craft in England at this time. Although this introduction contains overlays of "clerical" learning,

I am grateful to the Graduate School Office of Research and Projects of Southern Illinois University, Carbondale, for a grant which made possible the research for this paper. I should also like to thank Mr Alan Cohn and Miss Kathleen Eads of Morris Library at S.I.U. for their constant help in procuring books for my research through purchase or interlibrary loans. Finally, a special word of thanks is due Dr Stephen Victor of Simon Fraser University for several bibliographical leads and to Prof. John Hoag of the University of Colorado and Prof. Harold McFarlin of S.I.U. for their critiques of the paper while it was in draft form.

[1] H. R. Hahnloser, ed., *Villard de Honnecourt* (Vienna, 1935), p. 11: "Wilars de Honecort v(os) salue (et) si proie a tos ceus qui de ces engiens ouverront, c'on trovera en cest livre q(u)'il proient por s'arme (et) qu'il lor soviengne de lui. Car en cest livre puet o(n) trover grant consel de le grant force de maconerie (et) des engiens de carpenterie, (et) si troveres le force de le portraiture, les trais, ensi come li ars de iometrie le (com)ma(n)d(e) (et) ensaigne."

[2] *Ibid.*, p. 91: "Ci comence li force des trais de portraiture si con li ars de iometrie les ensaigne. por legierem(en)t ouvrer. (et) en l'autre fuel s(un)t cil d(e) le maconerie." What Villard could have meant by the "technique of the forms of masonry as taught by the art of geometry" will become clear, it is hoped, in the course of this paper.

it is patently founded on the traditions of the craft itself, and thus it provides a valuable insight into the masons' perceptions of the history and character of their craft.[3] The author begins with a review of the seven liberal arts, but he quickly singles out for special consideration geometry, which he defines in the old-fashioned way — following Isidore of Seville — as the measure of the earth. He then propounds the significance of geometry for the handicrafts:

> Marvel you not that I said that all science lives all only by the science of geometry. For there is no artifice nor handicraft that is wrought by man's hand but it is wrought by geometry. . . . For if a man works with his hands he works with some manner [of] tool, and there is no instrument of material things in this world but it comes of the kind of earth and to earth it will return again. And there is no instrument, that is to say, a tool to work with, but it has some proportion more or less. And proportion is measure, [and] the tool or the instrument is earth. And geometry is said [to be] the measure of earth, wherefore I may say that men live all by geometry.[4]

Having established the connection between geometry and work with tools, the author proceeds to say "that among all the crafts of the world of man's craft, masonry has the most notability and most part of this science [of] geometry, as it is noted and said in historial, as in the Bible and in the Master of Stories."[5] He next turns to the origins of geometry and masonry and recounts several versions drawn from these various sources. The one of greatest interest here is the story of Euclid, who, according to the author, had been a clerk of Abraham during the latter's sojourn in Egypt. Indeed, it was Abraham who had taught the science of geometry to Euclid, who in turn taught it to the Egyptians.

> Then this worthy clerk Euclid taught them to make great walls and ditches to hold out the water [of the Nile]. And he by geometry measured the land and departed it in divers parts, and made every man to close his own part with walls and ditches, and then it became a plenteous country. . . . And they took their sons to Euclid to govern them at his own will, and he taught to them the craft [of] masonry and gave it the name of geometry because of the parting of the ground that he had taught to the people.[6]

The chief value, for our present purposes, of this quaint and garbled account of the historical person Euclid, and of the origin and meaning of Euclidean geometry, is that it reveals the connotative significance which the word geometry had acquired for masons by 1400; in their view, masonry was a craft which historically went back to Abraham through Euclid, and which had originally been founded on that preeminent science of the handicrafts — geometry. For mediaeval masons Euclid had virtually become an eponymous hero of the craft, and the word geometry had become synonymous with masonry. When a word has acquired such rich and special meanings, we must beware of misreading it when

[3] See Douglas Knoop, G. P. Jones, and Douglas Hamer, eds., *The Two Earliest Masonic MSS. The* Regius MS. (*B.M. Bibl. Reg. 17 A1*); *The* Cooke MS. (*B.M. Add MS. 23198*), "Publications of the University of Manchester," No. CCLIX (Manchester, 1938), pp. 7–10, for discussion of the authorship and literary sources for the "Cooke MS.", to which I refer.

[4] *Ibid.*, pp. 73, 75. I have rendered this and later quotations from this text in modern spelling and punctuation.

[5] *Ibid.*, p. 75.

[6] *Ibid.*, pp. 95, 97.

used by those who have given it these meanings. That is to say, neither "Euclid" nor "geometry" may have meant to mediaeval masons what today we mean by Euclidean geometry.

To reconstruct the geometrical knowledge of mediaeval master masons, we must first consider the kind of education which these men would normally have obtained. Since in a previous study I have done this in detail for English master masons, only a summary review will be provided here.[7] It does not appear that literacy was a necessary accomplishment for a mason to become a master of his craft, for clerks were readily available to provide whatever reading and writing skills might be needed in the transaction of business and the keeping of records in building construction. On the other hand, there is evidence that from the thirteenth century onward at least some master masons learned to read and write; this seems to be simply part of the larger story of the increasing literacy of the laity in the later Middle Ages. Literacy in the vernacular languages could be acquired in a variety of ways — formal and informal — but literacy in Latin normally meant that one had attended a grammar school, where indeed the major thrust of the studies was to teach the young scholars to read, write, and speak Latin. Therein lies an important key to the problem of the geometrical knowledge of mediaeval master masons. While grammar school teachers normally paid lip service to the *quadrivium*, more often than not arithmetic, geometry, music, and astronomy received little attention in the actual curriculum. Thus even the lad who completed several years of study in a grammar school would have had little or no contact with Euclidean geometry, even in the diminished form in which it had come down through the early mediaeval encyclopedias, anthologies, and textbooks.

Only in the higher levels of formal schooling, that is, at some of the more renowned monastic and cathedral schools, the *studia generalia*, and later, the universities, could a student find geometry as a regular part of the curriculum.[8] But given the social, economic, and professional circumstances of mediaeval master masons, it may be inferred that a young man who wished to become a master in this craft would not have pursued such higher studies — and I have seen no evidence which contradicts this inference. Conversely, a young man who had studied at a university would expect to find career opportunities outside the building crafts themselves; he might become a clerk of the works, but not a master mason.

[7] Lon R. Shelby, "The Education of Medieval English Master Masons," *Mediaeval Studies*, XXXII (1970), 1–26.

[8] See Pearl Kibre, "The Quadrivium in the Thirteenth Century Universities (With Special Reference to Paris)," *Arts libéraux et philosophie au moyen âge*, "Actes du quatrième congrès international de philosophie médiévale (1967)," (Montréal, 1969), pp. 175–191; and James Weisheipl, "The Place of the Liberal Arts in the University Curriculum during the XIVth and XVth Centuries," *ibid.*, pp. 209–213. Unfortunately, in this Congrès' monumental program on the mediaeval *artes liberales*, virtually no attention was given to the place of the *trivium* and *quadrivium* in mediaeval grammar schools. The same is true for the compendium of studies in Josef Koch, ed., *Artes Liberales von der Antiken Bildung zur Wissenschaft des Mittelalters*, "Studien und Texte zur Geistesgeschichte des Mittelalters," Bd. v (Leiden, 1959).

It thus appears that whatever geometrical knowledge a mediaeval master mason might have possessed, he had not gotten it from formal schooling.[9] On the other hand, there were informal means for masons to acquire this knowledge — such as conversations with clerical building patrons that could amount to a kind of tutoring. And the really determined mason who was literate could teach himself geometry by studying the mediaeval treatises on the subject. But the deep conviction of the masons that "geometry" was the basis of their craft suggests that these informal — and probably quite exceptional — avenues do not satisfactorily explain how the rank and file of the masons, and even the master masons, acquired their knowledge of geometry. A far more probable answer is to be found in the masons' education into the traditions of their craft, whereby the technical knowledge required in design and construction was transmitted from father to son, from master to apprentice, from learned journeyman to those who were less learned in the craft traditions.[10] Since the geometry of the masons was an essential part of that technical knowledge, mediaeval master masons would normally have acquired their geometrical knowledge in the same way that they acquired the rest of their knowledge and skill in building — by mastering the traditions of the craft.

Those traditions were by and large transmitted orally from one generation of masons to the next; consequently, the vast bulk of the technical knowledge upon which mediaeval building and architecture were based disappeared with the dying of those oral traditions at the close of Gothic building in Europe. In view of this, the task of reconstructing the geometrical knowledge of mediaeval master masons would appear to be hopeless. Fortunately, however, near the end of the Middle Ages a few German master masons wrote little books on some of the technical aspects of their craft, and from these we can get a fairly substantial picture of the geometry of the masons. But before we turn to these late fifteenth-century documents we must give attention to that solitary and crucially important thirteenth-century *Sketchbook* of Villard de Honnecourt.

As noted above, Villard several times referred to the *ars de iometrie* as the basis for his technique of *portraiture*, and he implied that it was also the basis for the craft of *maconerie*. But since we have recognized the loaded character of the word geometry for mediaeval masons, we must ask, what precisely did Villard mean by *iometrie*? And what did his follower, Magister 2, mean by *geometrie* when he inscribed the sentence, "Totes ces figures sunt estraites de geometrie," on one of the pages devoted to *maconerie*? It has generally been assumed that Villard and Magister 2 meant "practical geometry" in the traditional mediaeval sense;

[9] This conclusion, based on my detailed study of the sources for education and the building crafts in mediaeval England, still corresponds to the views of Sigmund Günther on the mathematical education of laymen in the Middle Ages, set forth long ago in his *Geschichte des mathematischen Unterrichts im deutschen Mittelalter* (Berlin 1887), pp. 286–335; for geometry in particular see pp. 326–328. I am not aware of any detailed study of the mathematical education of laymen in mediaeval France or Italy.

[10] Shelby *op. cit.* pp. 18–23.

Magister 2's assertion has therefore been taken to mean, "All of these figures are drawn from [a treatise on practical] geometry."[11]

Certainly mediaeval scholars showed a considerable interest in practical geometry, as evidenced by the numerous extant treatises on the subject. If the geometry of the masons was even approximately equivalent to this practical geometry — if Magister 2 did copy his examples from a mediaeval *practica geometriae* — then we clearly have a wealth of material from which to reconstruct the geometrical knowledge of mediaeval masons, in addition to the *Sketchbook* of Villard and the booklets by late mediaeval German masters.

This is a critical problem for the subject of this paper. In order to deal with the specific meaning of Magister 2's assertion, and to provide a general framework for a discussion of the content of the geometry of mediaeval masons, it will be necessary at this point to make an excursus into the mediaeval traditions in geometry through the thirteenth century, with particular attention on the development of mediaeval practical geometry.[12] This does not require us to enter the vexed and much-discussed questions regarding the translations of Euclid's *Elements* and their dissemination in mediaeval Europe. Suffice it to say that during the early Middle Ages there were in circulation Latin translations (by

[11] Hahnloser *op. cit.*, pp. 196–197, 240–242, 254–259; and before him, Victor Mortet, "La mesure de la figure humaine et le canon des proportions d'après les dessins de Villard de Honnecourt, d'Albert Durer et de Léonard de Vinci," *Mélanges offerts à M. Émile Chatelain* (Paris, 1910), pp. 367–371.

[12] For the subject of mediaeval practical geometry the general histories of mathematics (F. Cajori, 1894; S. Günther, 1908; H. G. Zeuthen, 1912; D. E. Smith, 1923; J. E. Hofmann, 1953; H. W. Eves, 1953) and even the histories of geometry (M. Chasles, 1837; J. L. Coolidge, 1940; H. W. Eves, 1963) are of little value, with the exceptions of the following: Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, 3rd. ed. (Leipzig, 1907; rpt. Stuttgart, 1965), I, 821–878 and II, 35–53; Gina Loria, *Storia della matematiche*, Vol. I, *Antichità-Medio Evo-Rinascimento* (Turin, 1929), pp. 233–260; 393–396; and A. P. Juschkewitsch, *Geschichte der Mathematik im Mittelalter* (Basel, 1964), pp. 338–346 and 384–387. However, the best surveys of mediaeval practical geometry are two articles by Paul Tannery, "La géométrie au xi^e^ siècle," *Mémoires scientifiques*, ed. J. L. Heiberg *et. al.* (Paris and Toulouse, 1922), v, 79–102 [rpt. from *Revue générale internationale, scientifique, litteraire et artistique*, No. 15 (1897), 343–357]; and "Histoire des sciences: Géométrie," *Mémoires scientifiques* (Paris and Toulouse, 1930), x, 37–59 [rpt. from *Revue de synthèse historique*, II (1901), 283–299]. Also helpful is Victor Mortet, "Note historique sur l'emploi de procédés matériels et d'instruments usités dan la géométrie pratique au moyen âge (x^e^–xiii^e^ siècles)," *Congrès international de philosophie*, 2nd. session (Geneva, 1904), pp. 925–942. But the history of mediaeval geometry is yet to be written. The last two decades of the nineteenth century saw a spate of activity by German scholars — H. Weissenborn, S. Günther, J. L. Heiberg, and especially M. Curtze — and French scholars — C. Henry, V. Mortet, and P. Tannery — in publishing mediaeval texts and monographs on detailed problems. Curtze had undertaken to write a history of mediaeval geometry that would synthesize this work, but the project remained unfinished at his death in 1903. The death of Tannery in 1904, of Mortet in 1914, and the coming of W. W. I (which saw the demise of several German journals devoted to the history of mathematics) virtually brought to a close this concerted interest in mediaeval geometry. Fortunately, since W. W. II interest in the subject has been revived by Marshall Clagett, John Murdoch, Guy Beaujouan, Roger Baron, and H. L. L. Busard, as will become evident from the numerous citations to their work in this present study. We may also welcome into the field Prof. Murdoch's student, Stephen Victor, whose Harvard dissertation (1971) contains an edition and translation of a late twelfth-century *practica geometriae* which begins "Artis cuiuslibet consummatio. . . ."

Boethius, "Pseudo-Boethius," and others) of portions of the *Elements* which transmitted to western students some of the definitions, postulates, axioms, and propositions of Euclid, but without the Euclidean proofs.[13] Then in the twelfth century the entire *Elements* was translated into Latin from Arabic versions by at least three Latin scholars.[14]

Geometry came to mediaeval Europe not only by way of Euclidean fragments. Since late antiquity it had been recognized as one of the seven liberal arts, so that it received proper obeisance — but short shrift — in the handbooks of Martianus Capella, Cassiodorus Senator, and Isidore of Seville.[15] Furthermore, early mediaeval scholars who wanted a more detailed knowledge of practical geometry than was provided in the handbooks and "Boethian" excerpts from Euclid could turn to the treatises on surveying written by the Roman *agrimensores*. Fragments from these works of Frontinus, Hyginus, Balbus, Nipsus, Epaphroditus, Vitruvius Rufus, and others were preserved in the famous "Codex Arcerianus," a very early (sixth or seventh century) manuscript known to have been in the monastery at Bobbio in the tenth century.[16] But it is doubtful that early mediaeval scholars were interested in or capable of doing much more than copying these passages on geometry from the handbooks, the Euclidean excerpts, and the agrimensorial treatises. Not until the time of Gerbert of Reims (c. 940–1003) was a western Latin scholar able to understand these sources sufficiently to attempt a geometrical treatise on his own. To be sure, the compilation which came to be known as the *Geometria Gerberti* was not all written by Gerbert. Again, it is not necessary here to enter the complex questions regarding the authorship of this work.[17] Our purpose will be served by noting the achievement represented

[13] The best guide to the mediaeval MSS and to the modern editions and literature on the texts is Menso Folkerts' introductory chapters to his edition of one of the "Pseudo-Boethius" geometries: "*Boethius*" *Geometrie II: Ein mathematisches Lehrbuch des Mittelalters*, "Boethius: Texte und Abhandlungen zur Geschichte der exakten Wissenschaften," Bd. IX (Wiesbaden, 1970), pp. 3–107.

[14] Marshall Clagett, "The Medieval Latin Translations from the Arabic of the *Elements* of Euclid, with Special Emphasis on the Versions of Adelard of Bath," *Isis*, XLIV (1953), 16–30. Cf. John E. Murdoch, "The Medieval Euclid: Salient Aspects of the Translations of the *Elements* by Adelard of Bath and Campanus of Novara," *Revue de synthèse*, LXXXIX (1968), 67–74. On a twelfth-century translation of the *Elements* from Greek into Latin see *idem*, "Euclides Graeco-Latinus: A Hitherto Unknown Medieval Latin Translation of the *Elements* Made Directly from the Greek," *Harvard Studies in Classical Philology*, LXXI (1966), 249–270.

[15] *Martiani Capellae de Nuptiis Philologiae et Mercurii*, Lib. VI, ed. F. Eyssenhardt (Leipzig, 1866), pp. 194–254; *Cassiodori Senatoris Institutiones*, Lib. II, cap. vi, ed. R. A. B. Mynors (Oxford, 1937), pp. 150–153; *Isidori Hispalensis Episcopi Etymologiarum sive Originum Libri XX*, Lib. III, cap. viii–xiv, ed. W. M. Lindsay (Oxford, 1911). Cf. William H. Stahl, *Roman Science: Origins, Development, and Influence to the Later Middle Ages* (Madison, 1962), pp. 173, 177, 206–209, 216–218.

[16] For over a century modern scholars have wrestled with the complex problems of establishing and interpreting the texts of the *agrimensores*. For a guide to the literature on the subject and an interesting argument that the monastery of Corbie "was the gromatic and geometric capital of the mediaeval world," see B. L. Ullman, "Geometry in the Mediaeval Quadrivium," *Studi di bibliografia e di storia in onore di Tammaro de Marinis* (Verona, 1964), IV, 263–285.

[17] Nicolaus Bubnov, ed., *Gerberti postea Silvestris II papae opera mathematica* (Berlin, 1899; rpt. Hildesheim, 1963), pp. 310–313, summarized the scholarship on the problem and explained his own separation of the authentic *Geometria Gerberti* (edited on pp. 48–97) from the *Geometria incerti auctoris*

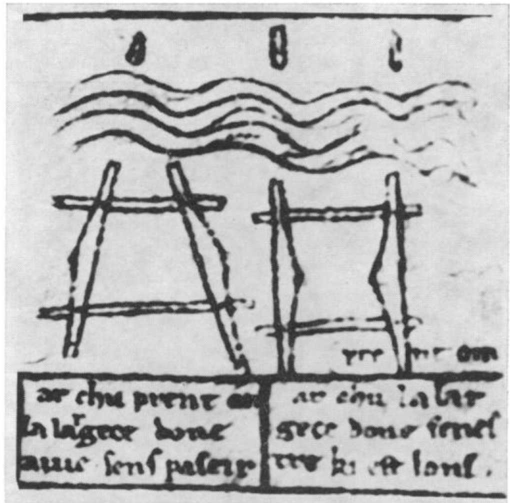

Fig. 1. Surveying technique, from Villard de Honnecourt's *Sketchbook* (ed. Hahnloser, Pl. 39 l, m).

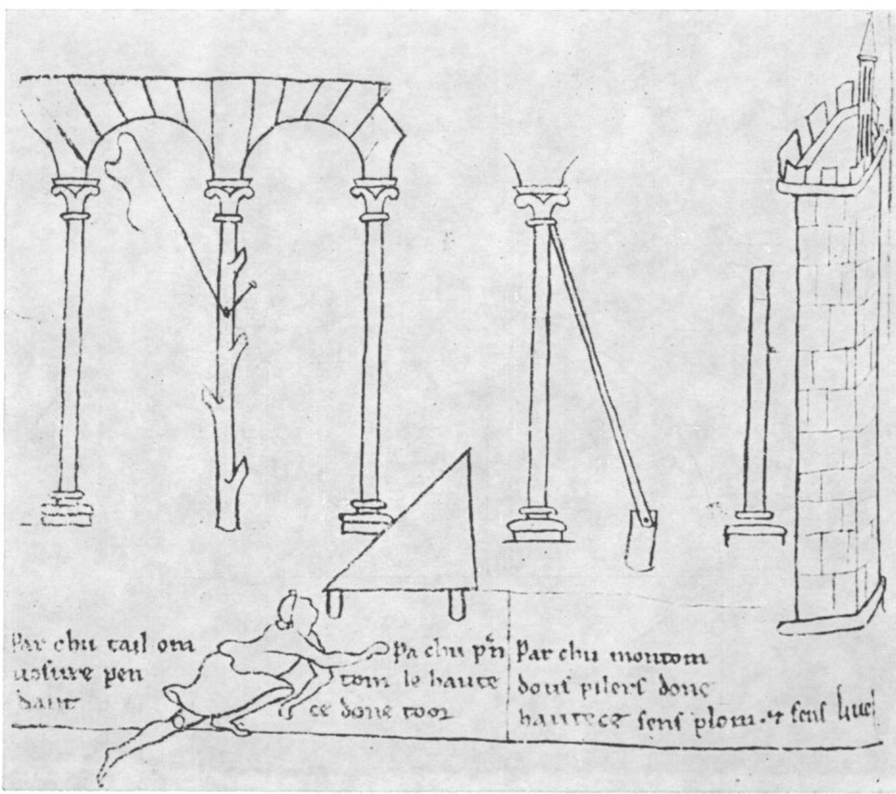

Fig. 2. Surveying technique, from Villard de Honnecourt's *Sketchbook* (ed. Hahnloser, Pl. 40 l).

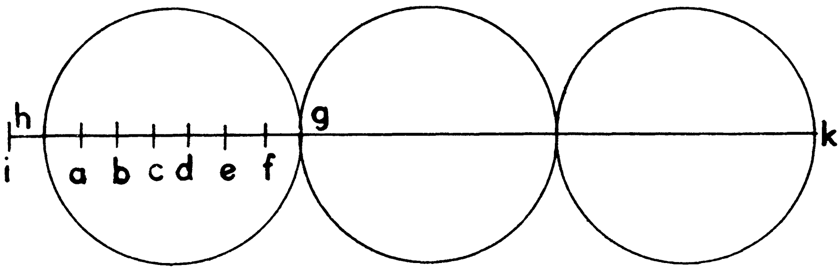

Fig. 3. Finding the circumference of a circle, redrawn from Roriczer's *Geometria deutsch.*

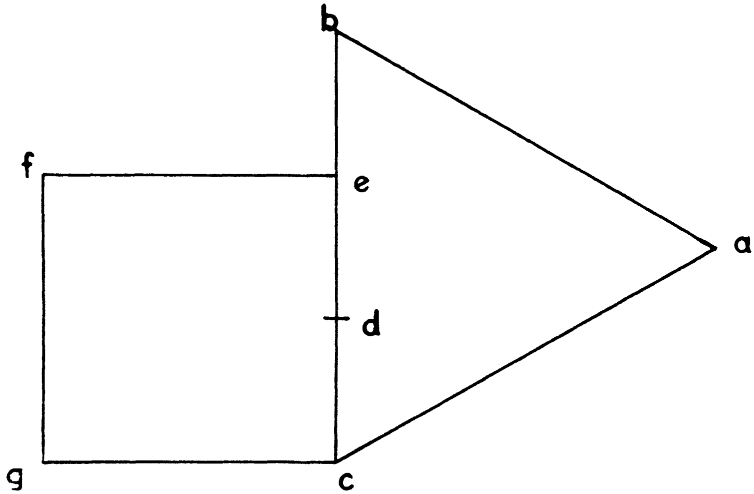

Fig. 4. Finding a square and triangle with the same area, redrawn from Roriczer's *Geometria deutsch.*

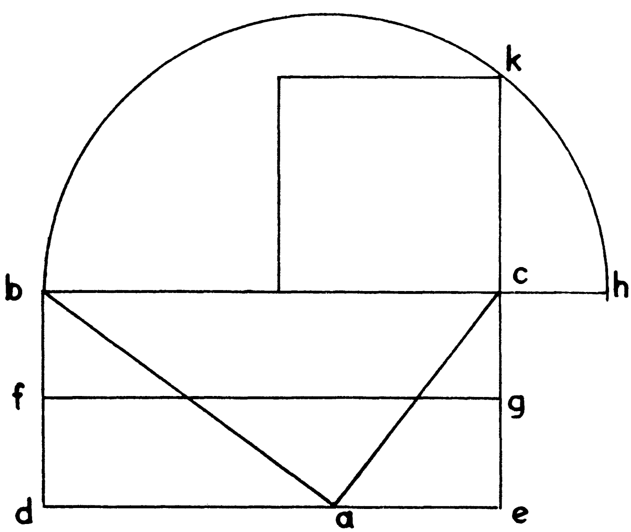

Fig. 5. Finding a square and triangle with the same area, redrawn from the anonymous *De inquisicione capacitatis figurarum.*

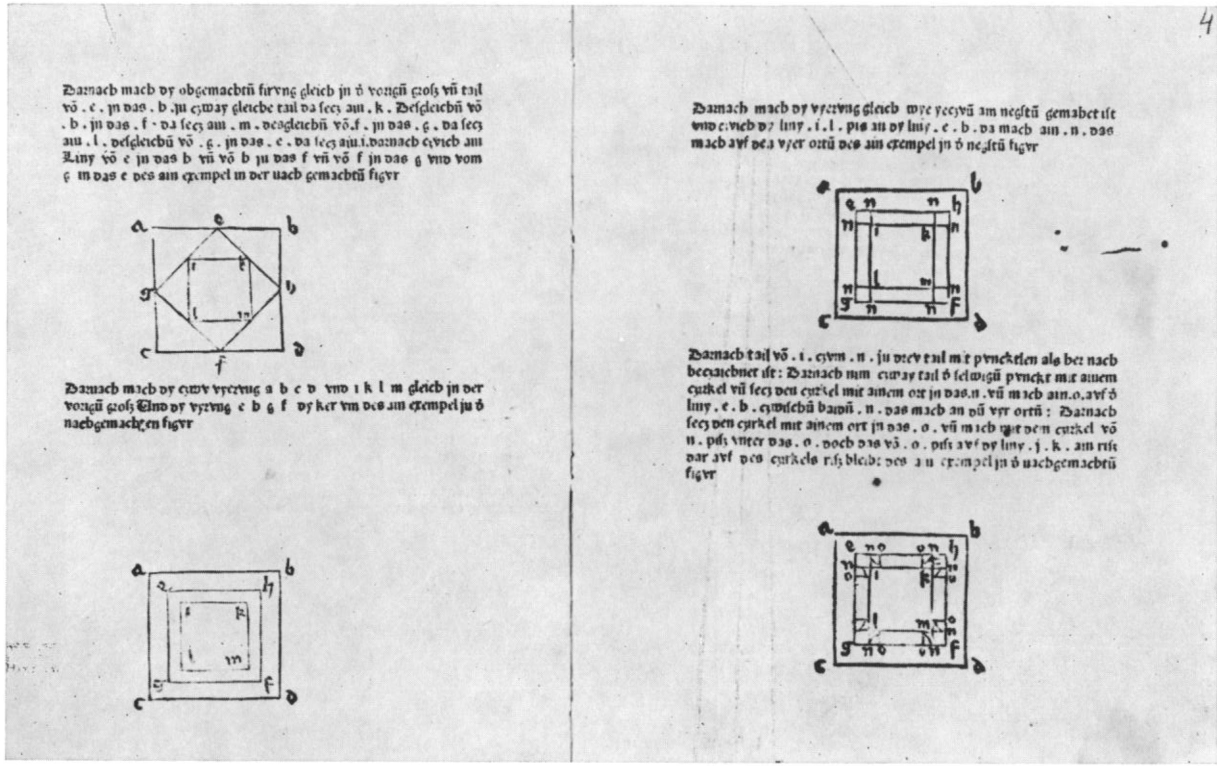

Fig. 6. Setting out the plan of a pinnacle, from Roriczer's *Büchlein von der Fialen Gerechtigkeit* (Würzburg, Universitätsbibliothek, I.t.q. XXXX, fols. 3ᵛ–4).

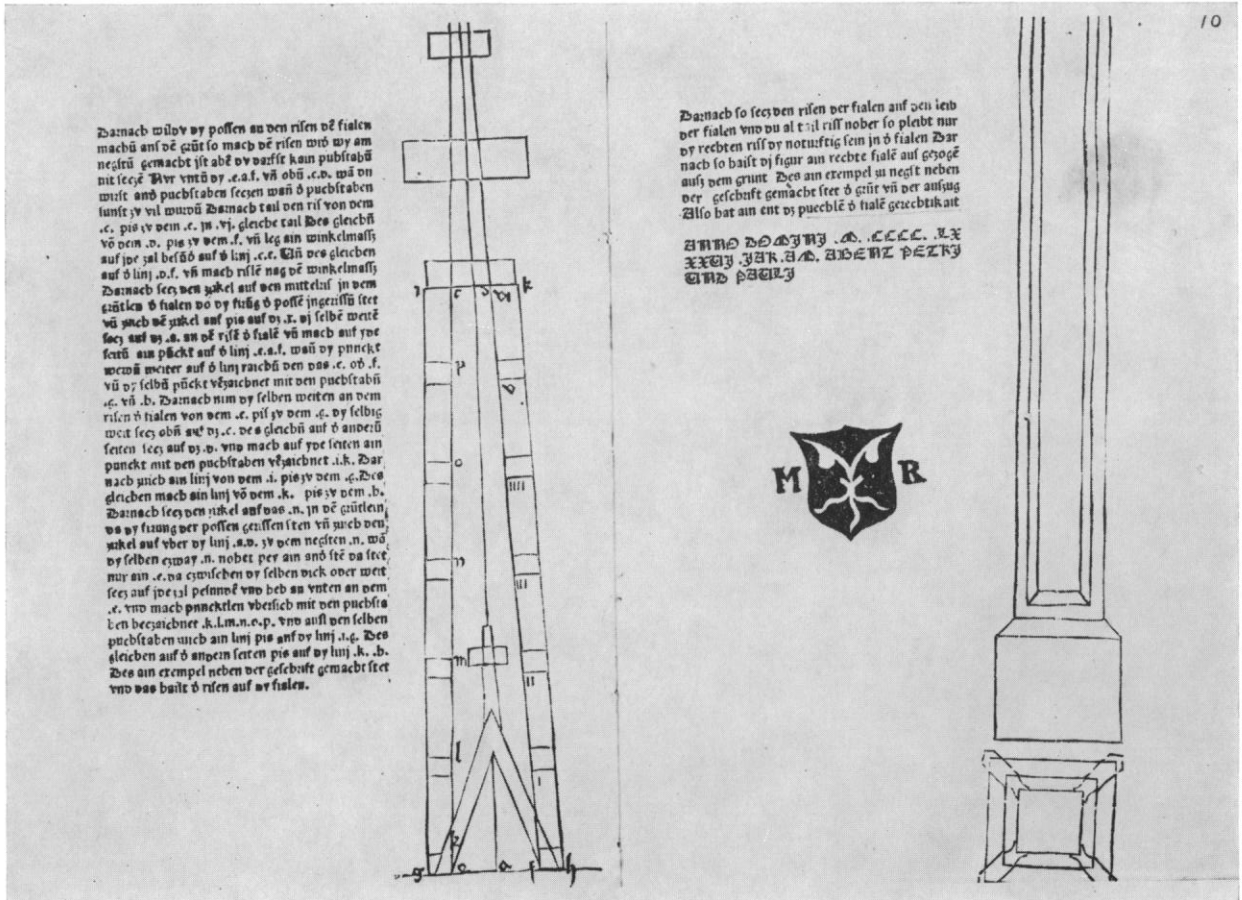

Fig. 7. Setting out the elevation of a pinnacle, from Roriczer's *Büchlein von der Fialen Gerechtigkeit* (Würzburg, Universitätsbibliothek, I.t.q. XXXX, fols. 9v–10).

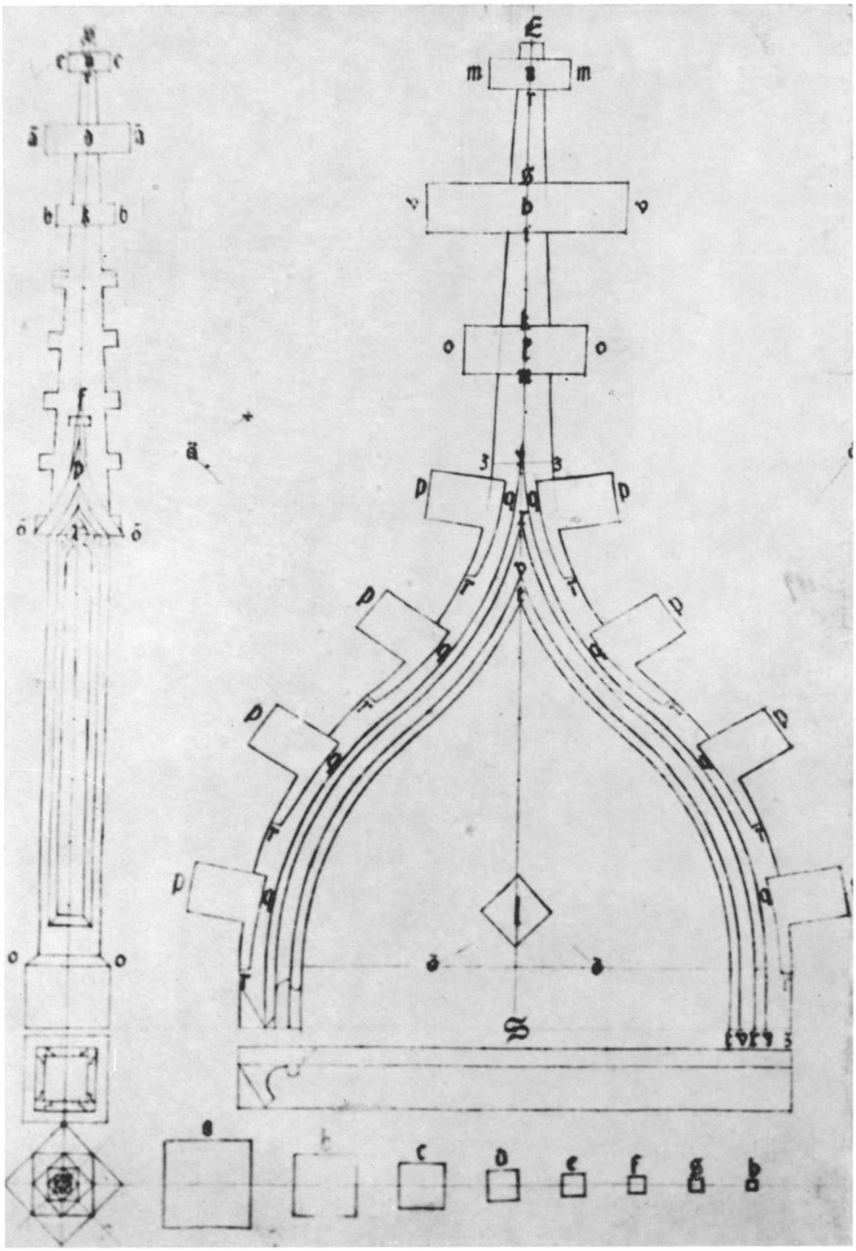

FIG. 8. Geometrical construction of a pinnacle and gable, from Schmuttermayer's *Fialenbüchlein* (Nürnberg, Germanisches Nationalmuseum, Bibliothek, No. 36,045, fol. 1v).

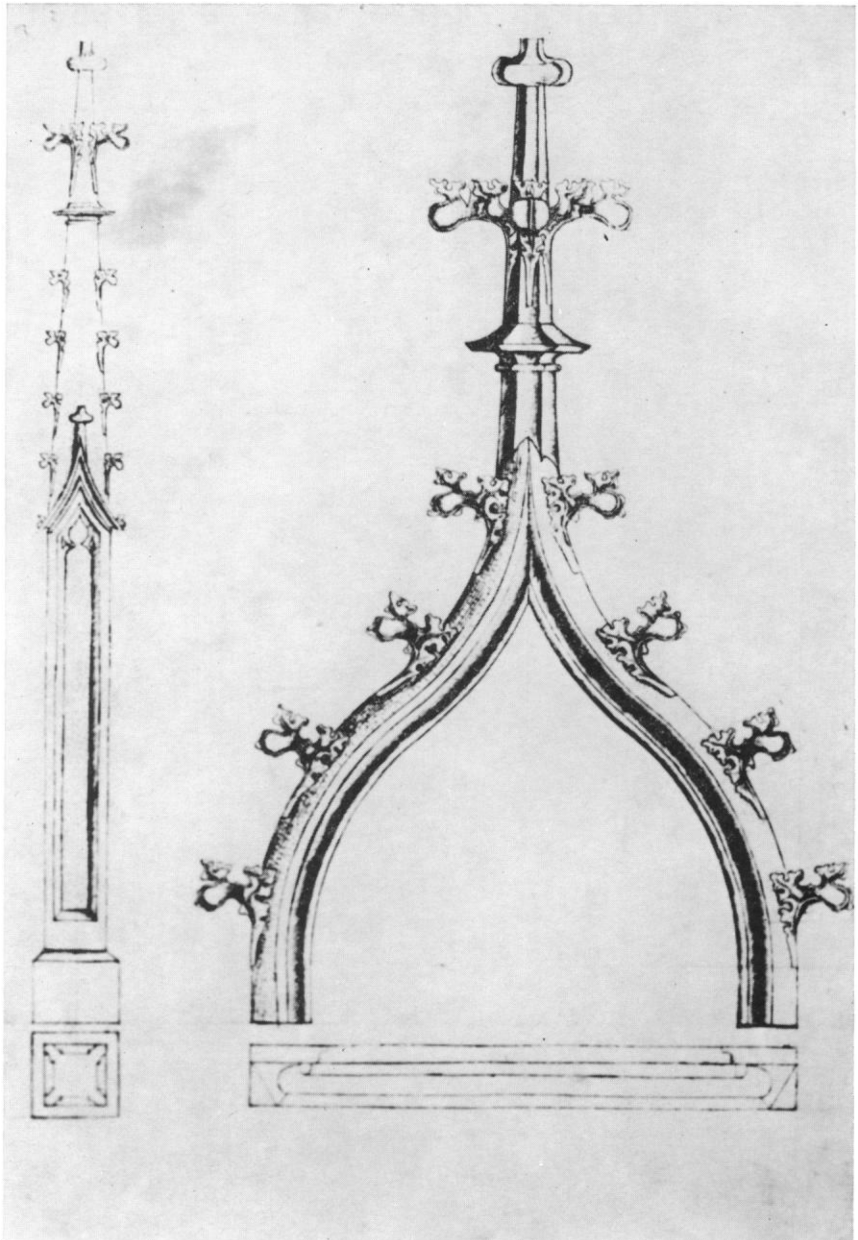

FIG. 9. Architectural rendering of a pinnacle and gable, from Schmuttermayer's *Fialenbüchlein* (Nürnberg, Germanisches Nationalmuseum, Bibliothek, No. 36,045, fol. 6).

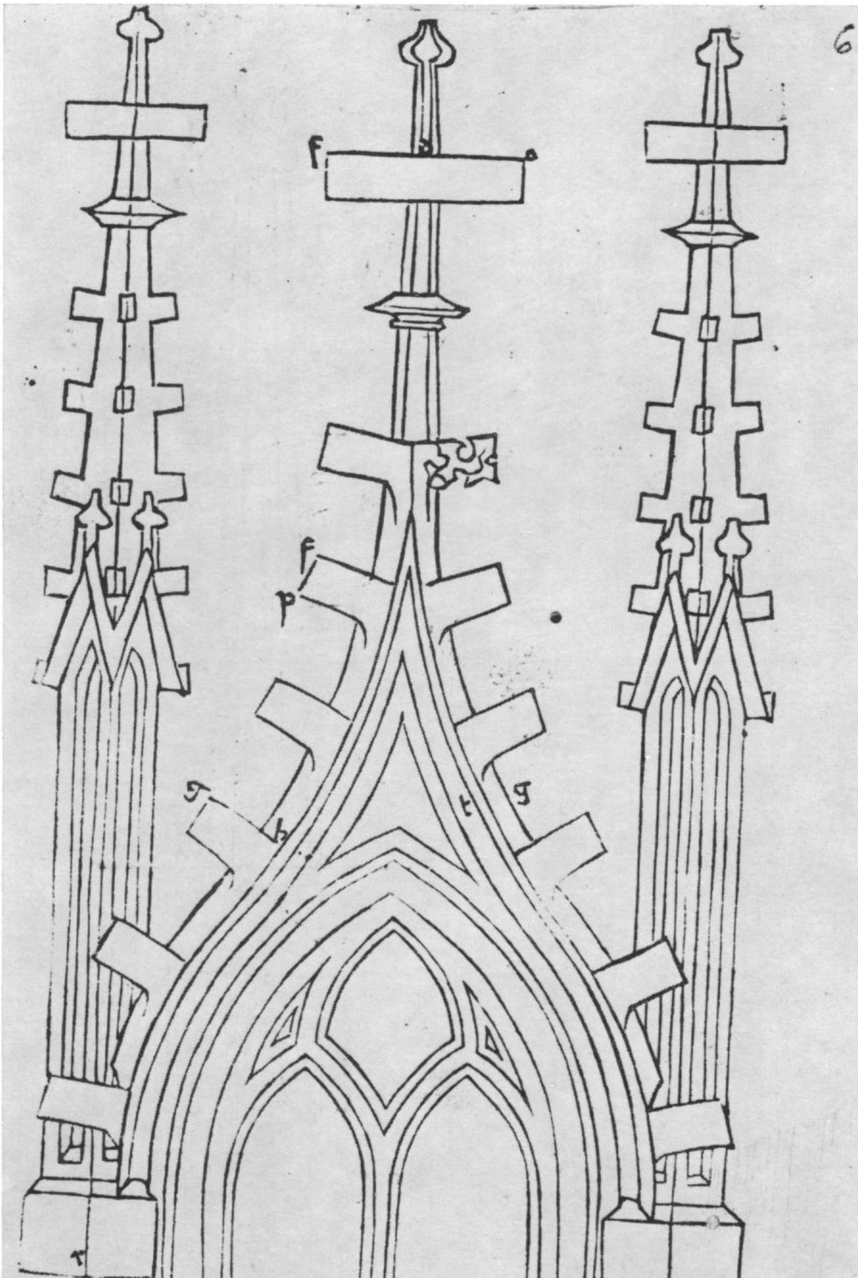

FIG. 10. Geometrical construction of a gable, from Roriczer's *Geometria deutsch* (Würzburg, Universitätsbibliothek, I.t.q. XXXX, fol. 6).

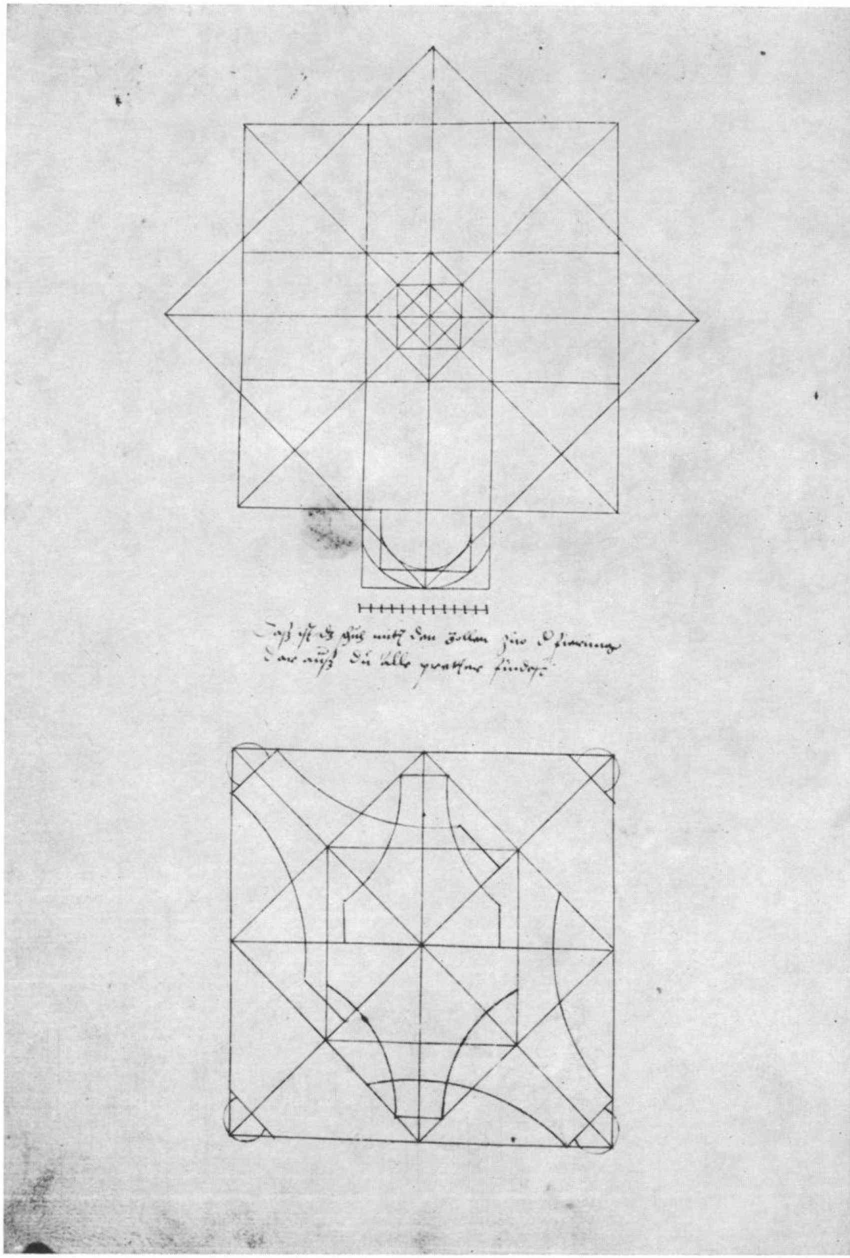

Fig. 11. Geometrical construction of a template for a window mullion, from Lechler's *Unterweisung* (Cologne, Historisches Archiv, Handschrift Wf. 276*, fol. 42v).

in this compilation by Gerbert and his eleventh-century successors, namely, the mastery of the Euclidean excerpts and the agrimensorial treatises at least to the extent that the authors could restate and even attempt to move beyond these sources in the formulation of geometrical problems. In reworking these materials the *Geometria Gerberti* provided the prototypes for the two main approaches to geometry in the High Middle Ages. The more strictly mathematical approach eventually found its way into a rather small corner of the university curriculum as the study of Euclid's *Elements*; practical geometry, on the other hand, became a subject of common interest to both Schoolmen and craftsmen, and treatises on it in Latin and in the vernacular languages continued to be written throughout the remainder of the Middle Ages.

But the formal distinction between "theoretical" and "practical" geometry did not appear in the Latin West until the twelfth century, when it was introduced by Hugh of St. Victor in a short treatise on *Practica geometriae*.[18] "The entire discipline of geometry is either theoretical, that is, speculative, or practical, that is, active. The theoretical is that which investigates spaces and distances of rational dimensions only by speculative reasoning; the practical is that which is done by means of certain instruments, and which makes judgments by proportionally joining together one thing with another."[19] Having separated theoretical from practical geometry, Hugh then developed a tripartite division of the latter into *altimetria*, *planimetria*, and *cosmimetria*. "Altimetry is that which investigates heights and depths. . . . It is called planimetry when one seeks to find the extent of a plane. Cosmimetry however takes its meaning from the word cosmos. Cosmos in Greek means the world; hence cosmimetry is the measurement of the world, that is to say, it concerns the measurement of circumference, as in the motion of a heavenly sphere and of other heavenly circles, or in the globe of the earth and many other things which nature has placed in the round."[20]

(edited on pp. 317–364). Tannery remained unconvinced of Bubnov's solution, however, and insisted that there were at least three authors involved in the compilation of this work. See his "Géométrie au xi^{e} siècle," pp. 99–101; and "Histoire des sciences: Géométrie," pp. 49–51.

[18] Roger Baron, "Sur l'introduction en Occident des termes 'geometria theorica et practica'," *Revue d'histoire des sciences et de leurs applications*, VIII (1955), 298.

[19] *Hvgonis de Sancto Victore opera propaedevtica: Practica geometriae, De grammatica, Epitome Dindimi in philosophiam*, ed. Roger Baron. "Publications in Mediaeval Studies," XX (Notre Dame, 1966), p. 16: ". . . omnis geometrica disciplina aut theorica est, id est speculatiua, aut practica, id est actiua. Theorica siquidem est que spacia et interualla dimensionum rationabilium sola rationis speculatione uestigat, practica uero est que quibusdam instrumentis agitur et ex aliis alia proportionaliter coniciendo diiudicat."

[20] *Ibid.*, p. 17: "Hinc namque altimetria dicta est quod sublime siue profundum uestigat. . . . Planimetria appellata uidetur quando porrectionem secundum planum persequitur. Cosmimetria autem ab eo quod et cosmus nomen accepit. Cosmus enim grece mundus dicitur, et inde cosmimetria dicta est quasi mensura mundi, ea uidelicet que circumferentiam metitur, quam in ambitu celestis spere et reliquorum circulorum celestium nec non in globo terre, multorum etiam aliorum que natura in orbem disposuit, consideramus." Cf. Hugh's definitions of altimetry, planimetry, and cosmimetry in his *Didascalicon*, Lib. II, cap. 14 [Migne, *Patrologia Latina*, CLXXVI, col. 757; and *The* Didascalicon *of Hugh of St. Victor*, trans. Jerome Taylor (New York, 1961), p. 70]. Damien van den Eynde, *Essai*

In short, Hugh's *Practica geometriae* is a Schoolman's textbook on surveying, both terrestrial and celestial. Hugh mentions in his "Prologue" that he is not attempting something new, but is simply pulling together material scattered through older works, in order to smooth the way for students interested in such matters. But what he did in effect was to establish a genre of scholastic treatises and to give to the genre its basic framework in the distinctions between altimetry, planimetry, and cosmimetry. Henceforth the mediaeval treatises on *practica geometriae* would normally follow the path marked out by Hugh, except that cosmimetry in time was transformed into stereometry. Thus mediaeval practical geometry as reflected in the treatises on the subject was confined to surveying and metrology, which meant that other applications of geometry to the world of practice remained outside the ken of the authors and readers of the *practicae geometriae*.

It might have been otherwise if these authors had taken up the suggestions on practical geometry which Dominicus Gundissalinus, the twelfth-century Spanish philosopher and translator, introduced into his schematization of knowledge, *De divisione philosophiae*. In this work Gundissalinus leaned heavily on the classification of the sciences developed by the tenth-century Arabic scholar, al-Farabi.[21] But he also appears to have been influenced by Hugh of St. Victor, for his discussion of geometry reflects a blending of terms and ideas from both the *Practica geometriae* and al-Farabi's *De scientiis*.[22] Following the latter's distinction between the theoretical and the practical sciences, Gundissalinus placed mathematics within the theoretical branch. He then divided mathematics into seven arts, one of which was geometry.[23] He further divided each of these arts into its

---

sur la succession et la date des écrits de Hugues de Saint-Victor, "Spicilegium Pontificii Athenaei Antoniani," XII (Rome, 1960), pp. 47–48, suggests that Hugh wrote the passage on geometry in the *Didascalicon* before he wrote the *Practica geometriae;* Roger Baron, "Note sur la succession et la date des écrits de Hugues de Saint-Victor," *Revue d'histoire ecclésiastique*, LVII (1962), 110, argues to the contrary. In either case, Hugh remains the source for this fundamental structuring of mediaeval *practicae geometriae*.

21 Gundissalinus himself translated al-Farabi's *De scientiis* and *De ortu scientiarum*. The former has been published in al-Farabi, *Catálogo de las ciencias*, ed. and trans. Ángel Gonzalez Palencia, 2nd. ed. (Madrid, 1953), pp. 87–115 (on geometry, see pp. 98–99); the latter in al-Farabi, *Über den Ursprung der Wissenschaften* (*De ortu scientiarum*), ed. Clemens Baeumker. "Beiträge zur Geschichte der Philosophie des Mittelalters," Bd. XIX, Heft 3 (Münster, 1916), pp. 17–24 (on geometry, p. 18).

22 Since my purpose is simply to outline the development of mediaeval *practicae geometriae* I shall not attempt here to sort out precisely what Gundissalinus owed to al-Farabi, to Hugh of St. Victor, and to his own thinking. A beginning in this task has been made by Roger Baron, "Note sur les variations au xii[e] siècle de la triade géométrique: Altimetria, Planimetria, Cosmimetria," *Isis*, XLVIII (1957), 31–32; but see R. W. Hunt's earlier attempt to find some of the intellectual roots of the *De divisione philosophiae* in the works of Thierry of Chartres: R. W. Hunt, "The Introductions to the 'Artes' in the Twelfth Century," *Studia Mediaevalia in honorem admodum Reverendi Patris Raymundi Josephi Martin* (Bruges, n.d. [1948]), pp. 86–93.

23 "Mathematica quoque uniuersalis est, quia sub ea continentur septem artes, que sunt arismetica, geometria, musica et astrologia, sciencia de aspectibus, sciencia de ponderibus, sciencia de ingeniis." Dominicus Gundissalinus, *De divisione philosophiae*, ed. Ludwig Baur. "Beiträge zur Geschichte der Philosophie des Mittelalters," Bd. IV, Heft 2–3 (Münster, 1903), pp. 31–32.

theoretical and practical aspects, and in good scholastic fashion he developed distinctions between the theoretical and the practical in terms of genus, species, parts, artificer, instruments, office, purpose, etc. Some of the distinctions which he made concerning geometry are particularly pertinent to our present study and worth quoting at length.

There are three species of practical geometry: altimetry, planimetry, cosmimetry. That science by which one considers lines, surfaces, and bodies in height is called altimetry, that is to say, the science of measuring altitudes; in planes it is called planimetry, that is, the science of measuring any plane surface; in depth, it is called cosmimetry or the science of measuring solids. . . . The purpose of theory is to teach something. The purpose of practice is to do something. . . . The artificer of theory is the geometer, who clearly knows all parts of geometry and can teach it. His instrument is the demonstration. . . . The artificer of practice is he who uses [geometry] in working. There are two kinds of these, namely, surveyors and craftsmen. Surveyors are those who measure the height and depth and plane surface of the earth. Craftsmen are those who exert themselves by working in the constructive or mechanical arts — such as the carpenter in wood, the smith in iron, the mason in clay and stones, and likewise every artificer of the mechanical arts — according to practical geometry. Each indeed forms lines, surfaces, squares, circles, etc., in material bodies in the manner appropriate to his art. These many kinds of craftsmen are distinguished according to the different materials in which and out of which they work. Any one of these thus has his proper materials and instruments. The instruments of the surveyors are the foot, palm, cubit, stadium, perch, and many others. Those of the carpenters are the axe, adze, broadaxe, string, and many others. Those of the smith are the anvil, shears, hammer, and many others. Those of the masons are the string, trowel, plumb bob, and many others. . . . The office of practical geometry is, in the matter of surveying, to determine the particular dimensions by height, depth, and breadth; in the matter of fabricating, it is to set the prescribed lines, surfaces, figures, and magnitudes according to which that type of work is determined. The goal is either the certification of dimensions, or money and praise for the completion of the work.[24]

It is clear from this passage that Gundissalinus followed Hugh of St. Victor's distinction between theoretical and practical geometry, but he significantly

[24] *Ibid.*, pp. 107–110: "Species quoque practice sunt tres: altimetria, planimetria, cosmimetria. Sciencia enim, qua considerat lineas superficies et corpora in altum, altimetria dicitur, scilicet sciencia de mensura altitudinis. qua uero in planum, dicitur planimetria i.e. scientia de mensura alicuius planiciei. qua uero in profundum, dicitur cosmimetria quasi sciencia de mensura solidi. . . . Finis enim theorice est aliquid docere. finis uero practice est aliquid agere. . . . Artifex uero theorice geometer, qui plane nouit omnes partes geometrie et eam docet. Instrumentum eius est demonstracio. . . . Artifex uero practice est, qui eam operando exercet. Duo autem sunt, qui eam operando exercent, scilicet mensores et fabri. mensores sunt, qui terre altitudinem uel profunditatem uel planiciem mensurant. fabri uero sunt, qui in fabricando siue in mechanicis artibus operando desudant, ut carpentarius in ligno, ferrarius in ferro, cementarius in luto et lapidibus et similiter omnis artifex mechanicarum arcium secundum geometriam practicam. Ipse enim per semetipsum format lineas, superficies, quadraturas, rotunditates et cetera in corpore materie, que subiecta est arti sue. horum autem fabrorum multe species esse dicuntur secundum diuersitatem materiarum in quibus et ex quibus operantur. quorum unusquisque sicut habet materiam propriam sic et instrumenta propria. mensorum enim instrumenta sunt pes, palmus, cubitus, stadium, pertica et multa alia. carpentariorum uero securis et ascia, dolabrum et linea et multa alia. ferrariorum uero incus, forficus et malleus et multa alia. cementariorum uero linea, trulla, perpendiculum et multa alia. . . . Officium uero practice est uel in mensurande de alto, profundo et lato certam dimensionem reddere, uel in fabricando statutas lineas, superficies, figuras et magnitudines secundum quod species operis exigit non excedere. Finis eius est uel certitudo dimensionis, uel merces et laus de consummacione operis."

broadened Hugh's definition of the latter. Whereas Hugh had limited his analysis of practical geometry to terrestrial and celestial surveying, Gundissalinus recognized that this "art" was used by *fabri* as well as *mensores*. Indeed, his perception of the essence of the geometry of the craftsmen was incisive; as we can see from the quotation above, that geometry consisted of the manipulation of "lines, surfaces, figures, and magnitudes" in the materials, with the instruments, and according to the rules appropriate for each craft. It is unfortunate that Gundissalinus' recognition of the *geometria fabrorum* was not followed up by mediaeval authors of *practicae geometriae*, for even a Schoolman's rendering of the practice of geometry by the building crafts would be a welcome source of information on a subject for which there is so little direct evidence.

There is virtually a conscious exclusion of the geometry of the craftsmen from the *Practica geometriae* completed by the now-famous Leonardo Pisano in 1220. Perhaps because this was an advanced treatise on mathematics, Leonardo did not feel obliged to begin with the usual distinctions between theoretical and practical geometry. Instead, he set down definitions for many of the terms and concepts with which he would be dealing, and then he launched forth on an eight-part treatise concerned with technical problems in geometry, arithmetic, trigonometry — and surveying. He devoted many pages to this latter subject in his large book, but they differed radically from the agrimensorial treatises and previous mediaeval works on surveying. Unlike the authors of those treatises, Leonardo did not merely provide rule-of-thumb procedures in surveying; he offered demonstrations of the mathematical correctness of the procedures which he described.[25] Thus Leonardo turned the mediaeval distinction between theoretical and practical geometry on its head, for in his *Practica geometriae* he demonstrated with "theoretical" geometry the proofs of his propositions. But he paid a price for his mathematical sophistication, since his book does not appear to have been widely read or used by others interested in practical geometry.

It was not only the sophistication of Leonardo's mathematics that put his book beyond the reach of craftsmen; unless they had at least a pretty thorough grammar school education, they would have been unable to understand the Latin in which it was written. The same of course would have been true of Hugh of St. Victor's *Practica geometriae*. While Latin was a great boon to scholarship because of its universality, it did constitute a stumbling block for those laymen who did not know the language, but who wanted direct access to at least some of the learning of the Schoolmen. The increasing literacy of the laity in the

[25] Leonardo made his intentions clear in his preface: "Rogasti amice dominice et reverende magister, ut tibi librum in pratica geometrię conscriberem; igitur amicitia tua coactus, tuis precibus condescens, opus iam dudum inceptum taliter tui gratia edidi, ut hi qui secundum demonstrationes geometricas: et hi qui secundum uulgarem consuetudinem, quasi laicalj more, in dimensionibus uoluerint operari super .viij. huius artis distinctiones, que inferius explicantur, perfectum inueniant documentum." *Scritti di Leonardo Pisano*, Vol. II, *Practica geometriae ed opuscoli*, ed. B. Boncompagni (Rome, 1862), p. 1. At another point he wisecracked: "Sed hec talis inuestigatio non est operando ab agrj mensoribus, qui secundum uulgarem modum procedure uolunt. . . . Sed ut ipsi, qui secundum geometricam scientam operarj desiderant leuius quam dictum sit. . . ." (p. 95)

vernacular languages in the later Middle Ages produced a certain amount of popular pressure for learned treatises to be written or translated into the vernacular. This desire extended to mathematical works as early as the thirteenth century, as may be noted from the introductory comment in a versified algorism dating from that period. "The two clerks who translated the computus into French are urged by many people to undertake the task of putting algorism into French, just as they did the computus. . . ."[26] Indeed, the number of treatises on "applied mathematics" composed in Latin and in the vernacular tongues from the thirteenth century onward reveals that here was a common ground of interest for Schoolmen and laymen alike. For an example one may cite another algorism in French, dating from c. 1275, but more pertinent to the present study is the French *Pratike de geometrie* to be found in the same manuscript.[27]

The anonymous author begins his *Pratike de geometrie* with this prefatory statement: "We shall commence a work on the practice of geometry, which we shall divide in three parts. In the first part we shall teach how to find the measurement of plane surfaces; in the second, how to find the measure of heights and depths and of large measures; in the third, how to find the details of geometry and astronomy appropriate to the two preceding parts."[28] One recognizes immediately the customary tripartite division of practical geometry into planimetry, altimetry, and cosmimetry — although the author appears a bit uncertain about what he is to do with that third section. He does not in fact follow his own outline very well, as may be ascertained from a brief review of the contents of the treatise.

First there is a short description of how to use the astrolabe in calculating the length of a straight line, for examples, the distance across a woods or a river, or the height of a tree or steeple. Next comes an explanation of how to find the area of various geometrical figures — the circle, square, pentagon, hexagon, heptagon, and a number of different triangles. This is followed by exercises in finding "surpluses," that is, the difference between the areas of a circle and of a square which inscribes it, and vice versa. The author then turns to some practical surveying problems, such as how to find the number of acres in a field, the number of messuages of a given size in a given area, the number of messuages in a round city. The last of the "geometrical" problems concerns the measurement of volumes of various containers, such as the hogshead and tun. The little treatise closes with

[26] Translation by E. G. R. Waters, "A Thirteenth Century Algorism in French Verse," *Isis*, XI (1928), 55.

[27] Charles Henry, "Sur les deux plus anciens traités français d'algorisme et de géométrie," *Bulletino di bibliografia e di storia delle scienze matematiche e fisiche*, XV (1882), 53–70, edited both little treatises. Victor Mortet, "Le plus ancien traité français d'algorisme," *Bibliotheca Mathematica*, Ser. 3, IX (1908–09), 60–63, re-edited the algorism and promised a new edition of the *Pratike de geometrie*, but the latter never appeared in print. Dr. Stephen Victor informs me that he is preparing a new edition; this is certainly desirable, for Henry's edition is quite unsatisfactory, as Mortet noted long ago.

[28] Henry, *op. cit.*, p. 55: "Nous commencerons une oeure soi le pratike de geometrie la ke le nous deuiserons en .3. parties. En la premiere partie ensengerons nous a trouer le mesure des planetes. En le seconde a trouer le mesure des hautes ches et des profondeces et des crasses mesures en la tierce a trouer les minuces de gyometrie et dastronomie couignables as .IJ. parties deuant."

several pages devoted to methods of calculating exchanges of money from one system to another.

This description may serve as an index of the contents of practical geometry as perceived by the anonymous author, and perhaps by those "many people" anxious to obtain works like this in the French language. In terms of applied geometry, clearly the author's main interests were directed towards surveying and metrology — the latter meaning the techniques of measuring the volume of various kinds of containers.[29] With the exception of the tunmaker, he scarcely bothers with the application of practical geometry to the crafts concerned with the mechanical and constructive arts. Only in one passage does he deal directly with a problem that might arise in building construction; there he provides a somewhat confused formula for finding the volume of a round column.[30] But even this formula probably did not derive as much from mediaeval as from classical practice, for it seems to be little more than a pale reflection of the fragment from antiquity, *De geometria columnarum et mensuriis aliis*, which had gotten included in some of the mediaeval manuscripts preserving the agrimensorial treatises and other fragments of Roman practical geometry.[31]

The *Pratike de geometrie* brings our review of mediaeval practical geometry back to Villard's *Sketchbook*, for the two books were approximately contemporary, they were both written in the Picard dialect, and they both were concerned with the application of geometry to practical problems. But generally speaking, the problems which interested Villard and Magister 2 were quite different from those which caught the attention of the author of the *Pratike de geometrie*, for the latter was concerned only with surveying and metrology, whereas the pages of the *Sketchbook* were primarily devoted to problems of *portraiture*, *maconerie*, and

[29] This thirteenth-century French author was thus carrying on the classical tradition of handbooks and treatises on metrology which applied practical geometry and arithmetic to the work-a-day problems of weights and measures. For the classical texts, see Friedrich Hultsch, ed., *Metrologicorum scriptorum reliquiae* (Leipzig, 1864–66), I, 179–355 and II, 48–146. Even the section on money-changing in the *Pratike de geometrie* was within the classical tradition, for this was an important branch of ancient metrology: Friedrich Hultsch, *Griechische und Römische Metrologie*, 2nd. ed. (Berlin, 1882), pp. 162ff.

[30] Henry, *op. cit.*, p. 61: "Se tu ueus trouer le coube dun piler reont tu troueras laire par le moitie de son dyametre en sa circonference. Keure laire sor le lonc la somme fera la coube du piler che pues prouer ausi ke deuant."

[31] Victor Mortet edited the fragment and made a detailed historical and philological analysis of the text in "La mesure des colonnes à la fin de l'époque romaine d'après un très ancien formulaire," *Bibliothèque de l'École des Chartes*, LVII (1896), 277–324; he followed this with a study of early mediaeval texts which showed the continuing literary influence of these classical formulae for mensuration and proportions of columns: "La mesure et les proportions des colonnes antiques d'après quelques compilations et commentaires antérieurs au xii^e siècle," *ibid.*, LIX (1898), 56–72. The formula for calculating the volume of a column was included in the *Geometria incerti auctoris*: Bubnov, *op. cit.*, p. 360. The traditional character of this problem is indicated by the fact that the simplified formula for its solution — very similar to that given in the *Pratike de geometrie* — was included in the section on stereometry in the *Tractatus quadrantis* composed by Robertus Anglicus sometime before 1276: Paul Tannery, ed., "Le Traité du quadrant de Maître Robert Anglès (Montpellier, xiii^e siècle). Texte latine et ancienne traduction grecque," *Mémoires scientifiques*, v, 188 [rpt. from *Notices et extraits des manuscrits de la Bibliothèque Nationale*, xxxv, pt. 2 (1897), 561–640].

carpenterie. To be sure, Villard and Magister 2 did include a few problems in surveying — how to measure the width of a river or of a window from a distance, or the height of a tower. (Figs. 1 and 2) The first and third of these were traditional problems of *altimetria* and *planimetria* in the agrimensorial and mediaeval literature on surveying. Indeed, Villard's demonstration of how to measure a tower could have served as an illustration for the description of this technique in the *Geometria Gerberti*:

The geometer devises a right-angle triangle composed of a base and vertical of the same number; the proportion of the hypotenuse is not considered, since in determining height by means of a right-angle triangle, it is thought to be quite useless. The device is carried by the surveyor along the base plane to the point where — with the eye placed at ground-level — the summit of the height to be investigated can be seen at the top of the vertical. Then, from the place where the view has been taken, one measures the base plane to the foot [of the thing which is being measured], and however far it is, that is the height.[32]

Conceivably, Villard got his knowledge of the technique directly from the *Geometria Gerberti*, although by his time it could well have become a rule-of-thumb practice handed down through the oral traditions of the craft. The traditional character of this rather crude technique is indicated by its survival in an English version of the fifteenth century, when surveying techniques had become even more refined than in the thirteenth century.[33] I think it doubtful that Villard extracted the technique directly from the *Geometria Gerberti*, but even if he did, it is important to note what he passed over in selecting this particular technique. For in fact this is only one, and the simplest, of several techniques for measuring heights which are described in the *Geometria Gerberti*. The others measure by means of a mirror, a shadow, a staff, a string and arrow, and an astrolabe, and some of the procedures are fairly complex.[34] Since neither Villard nor Magister 2 gives evidence of using either the astrolabe or the surveyor's quadrant, one suspects that they were not abreast of the advances in surveying which had come to Europe with the introduction of Arabic learning and mathematical instruments.[35]

[32] Bubnov, *op. cit.*, p. 327–328: "Componatur a geometra orthogonium basi cathetoque ejusdem numeri compositum, hypotenusae vero proportio praetermittatur, quae ad altum investigandum in hoc orthogonio prorsus inutilis judicatur. Compositum autem tandiu per planum a mensore trahatur, donec oculo humi apposito per catheti summitatem summitas altitudinis investigandae cernatur. Qua visa, a loco, cui visus inhaeserat, planities ad radicem usque metiatur; et quanta fuerit, tanta altitudo dicatur." This passage is from the eleventh-century *Geometria incerti auctoris*. Cf. Maximilian Curtze's edition of this section of the *Geometria Gerberti* in "Die Handschrift No. 14836 der Königl. Hof- und Staatsbibliothek zu München," *Abhandlungen zur Geschichte der Mathematik*, VII (1895), 84–95.

[33] J. O. Halliwell, ed., *Rara Mathematica* (London, 1839), pp. 27–28, transcribed this English document from the British Museum MS Lansdowne 762, fol. 23b. Leonardo Pisano also included the technique in his *Practica geometriae* (pp. 202–203 of Boncompagni's edition), but in typical fashion he provided a detailed mathematical explanation of the technique, as well as variations on it with the use of similar but non-equilateral triangles.

[34] See, for example, the procedure, "Ad rem inaccessibilem nobis altioribus metiendam," Bubnov, *op. cit.*, pp. 328–330.

[35] On mediaeval surveying instruments, see Maximilian Curtze, "Über die im Mittelalter zur Feldmessung benutzten Instrumente," *Bibliotheca Mathematica*, N.F., x (1896), 65–72; and R. T. Gunther, *Early Science in Oxford* (Oxford, 1921–23; rpt. London, 1967), I, 333–340. For knowledge

The question of the source of Villard's technique for measuring the height of a tower brings us back to the problem of Magister 2's assertion that "Totes ces figures sunt estraites de geometrie." He could hardly have meant that *all* of these practical problems of the masons' craft were drawn from some mediaeval treatise on practical geometry. As we have seen, these treatises were almost entirely confined to problems of surveying and metrology, and their authors showed little concern for the application of geometry to the building and mechanical crafts. If not from some *practica geometriae,* were the examples of practical problems taken from another shop-manual of the masons' craft, as suggested by Prof. Robert Branner?[36] In spite of the ingenuity with which Branner argues this possibility, it remains an *argumentum e silentio.* Apart from the *Sketchbook* itself, there is not a whit of evidence for the existence at this time of other shop-manuals of the masons' craft, let alone a continuing tradition of such books of which Villard's is the only survivor.[37]

There are so many puzzling problems in Villard's *Sketchbook* — and they have elicited such elaborate and diverse explanations — one is tempted to posit as a fundamental exegetical principle the rule that the simplest explanation is *ipso facto* the best. Therefore, instead of assuming that Magister 2 copied the shop problems on folios 39 and 40 from some other book — of whatever kind — let us begin with the fact that he inserted additional examples of practical problems of the masons' craft into those pages which Villard had already set aside for *maconerie.* Let us then assume that the figures and texts were not necessarily meant to be self-explanatory, and that Magister 2 inserted them as reminders, as memory tags, for other masters or journeymen who would orally explain the details of the problems and their solutions to apprentices and fellow masons of the craft. In brief, let us assume that the *Sketchbook* is what it appears to be, namely, an exceptional literary record of some of the oral traditions of the masons' craft. These simple assumptions help to explain why the *Sketchbook* contains so many unresolved, and perhaps in some cases unresolvable, puzzles; they also make it unnecessary to set up further puzzles about the relationship of this document to a completely unknown — and perhaps never existent — literary genre of the thirteenth century.

What then of Magister 2's assertion about the source of his figures? Within the context of an oral rather than written tradition, the statement would mean that these figures were drawn from the "science" of geometry upon which all the handicrafts are based, and which "masonry has the most notability and most part of," to borrow a phrase from the "Articles and Points of Masonry." And what is the content of that geometry? Since it is not that of the treatises on *practica geo-*

---

and use of the astrolabe and quadrant by the authors of twelfth- and thirteenth-century *practicae geometriae,* see *Hvgonis de Sancto Victore opera propaedevtica,* pp. 25ff.; Henry, "Anciens traités français," p. 55; Leonardo Pisano, *Practica geometriae,* pp. 204ff.; and Tannery, "Traité du quadrant," pp. 151ff.

[36] "A Note on Gothic Architects and Scholars," *Burlington Magazine,* XCIX (1957), 372–375.

[37] As suggested by Paul Frankl, *The Gothic: Literary Sources and Interpretations through Eight Centuries* (Princeton, 1960), pp. 47–48.

metriae, and since there are extant no other previous or contemporary shop manuals of the masons' craft, Magister 2's statement does not direct us outward to other literary documents, but inward to the *Sketchbook* itself, as the primary source for reconstructing the content of the geometry used by master masons of this period.

Concentrating first on the pages devoted to *maconerie*, we may enumerate the kinds of problems which Villard and Magister 2 proposed to solve with the geometry of the masons' craft. Of the thirty-eight problems on these three pages, four figures (39 l, m, 40 l, and 41 b of Hahnloser's edition) deal with techniques of surveying, while another three (39 j, p, and q) essentially belong to other crafts. Five figures (39 e, k, n, and 41 a, c) deal with larger problems of designing and setting out masonry work at full scale. The remainder are more strictly speaking stereotomical problems of setting out and cutting various stones used in the masons' repertoire of architectural forms: columns, springers, pendants, cusps, and above all, voussoirs of different kinds. Several generations of scholars have entertained themselves in trying to sort out the puzzles which these figures and cryptic comments present. In recent years Professor Branner and I have published a series of articles on these figures, and while we have sometimes differed on points of detail, our approach has been similar in seeking the simplest possible explanation of the techniques of the masons' craft to which the drawings refer.[38] Because of these studies, it will not be necessary in this paper to go into details of the stereotomical techniques of mediaeval masons as illustrated in Villard's *Sketchbook*. But we may note the general conclusion which has emerged from these studies, namely, that stereotomical problems were solved by mediaeval masons primarily through the physical manipulation of geometrical forms by means of the instruments and tools available to the masons. These were rule-of-thumb procedures, to be followed step by step, and there were virtually no mathematical calculations involved.

We may thus characterize the practical geometry of Villard's *Sketchbook* more precisely as constructive geometry, by means of which technical problems of design and building were solved through the construction and physical manipulation of simple geometrical forms — triangles, squares, polygons, and circles.

This reconstruction of the geometry of Villard and Magister 2 is based on the stereotomical problems of the pages devoted to *maconerie*; but once having defined it, one easily recognizes its application to other problems in the *Sketchbook*. For instance, the surveying techniques noted above now reveal themselves entirely as physical procedures. To find the height of a tower, a mason sets up an

[38] Robert Branner, "Three Problems from the Villard de Honnecourt Manuscript," *Art Bulletin*, XXXIX (1957), 61–66; *idem*, "Villard de Honnecourt, Archimedes, and Chartres," *Journal of the Society of Architectural Historians*, XIX (1960), 91–96 (with additional discussion in the "Letters to the Editor," *ibid.*, XX (1961), 143–146; *idem*, "Villard de Honnecourt, Reims and the Origin of Gothic Architectural Drawing," *Gazette des Beaux-Arts*, Ser. 6, LXI (1963), 129–146; Lon R. Shelby, "Medieval Masons' Tools. II. Compass and Square," *Technology and Culture*, VI (1965), 236–248; *idem*, "Setting Out the Keystones of Pointed Arches: A Note on Medieval *Baugeometrie*," *ibid.*, X (1969), 537–548; and *idem*, "Mediaeval Masons' Templates," *Journal of the Society of Architectural Historians*, XXX (1971), 140–154.

instrument in the shape of a right-angle isosceles triangle at a distance from the tower which allows the line of sight along the hypotenuse to strike the top of the tower. The distance from the instrument to the tower will then be the height of the tower. Very simple, very physical, and very non-mathematical. Although the procedure was based on a theorem of similar triangles, the mason did not have to know that; all he had to do was to follow the procedure correctly to obtain the height of the tower. One cannot tell from the drawing whether Villard himself understood the geometrical principle which underlay the procedure. In this respect, the difference between the "content" of the masons' geometry and even that of the *Pratike de geometrie* becomes evident, for though the latter also used simplified procedures, these did involve some mathematical calculations, as well as simple mathematical demonstrations of the correctness of the procedures described.

The very same kind of constructive geometry is to be found in those pages of the *Sketchbook* devoted to *portraiture*, where Villard taught the method of drawing according to the *ars de iometrie.* Again it is a matter of constructing simple geometrical forms that provided the framework into which, or around which, the drawing was devised. Professor Panofsky clearly perceived the essence of this constructive geometry in his classic study on "The History of the Theory of Human Proportions as a Reflection of the History of Styles":

> What the French architect Villard de Honnecourt wants to transmit to his *confrères* as the "art de pourtraicture" is a "méthode expéditive du dessin" which has but little to do with the measurement of proportions, and from the outset ignores the natural structure of the organism. Here the figure is no longer "measured" at all, not even according to head- or face-lengths; the schema almost completely renounced, so to speak, the object. The system of lines—often conceived from a purely ornamental point of view and at times quite comparable to the shapes of Gothic tracery — is superimposed upon the human form like an independent wire framework. The straight lines are "guiding lines" rather than measuring lines. . . .[39]

The *ars de iometrie* of Villard and Magister 2 was thus one and the same throughout the *Sketchbook*, whether it was applied to problems of drawing the faces and bodies of men and animals (Hahnloser, pls. 35–38), or to calculating the height of a tower (40 1), or to devising the shape of the keystones for third- and fifth-point arches (40 c, d),[40] or to delineating the plan of one level of a tower of Laon Cathedral (18 a).[41] In brief, the "art of geometry," as far as these masons were concerned, was "the technique of the forms" (*li force des trais*), just as Vil-

[39] Erwin Panofsky, *Meaning in the Visual Arts* (Garden City, N. Y., 1955), p. 83.

[40] See my article, "Setting Out the Keystones," especially pp. 541–545.

[41] See Walter Ueberwasser, "Nach Rechtem Masz: Aussagen über den Begriff des Maszes in der Kunst des XIII.–XVI. Jahrhunderts," *Jahrbuch der preussischen Kunstsammlungen*, LVI (1935), 259–261 and Abb. 7; and Maria Velte, *Die Anwendung der Quadratur und Triangulatur bei der Grund- und Aufrissgestaltung der gotischen Kirchen* (Basel, 1951), pp. 53–55 and Taf. VIII; both of whom apply the technique of quadrature to Villard's plan of the tower, though Ueberwasser begins with the outermost measurements, including the buttresses, while Velte's scheme excludes the buttresses. Neither scholar may be precisely correct in reconstructing the procedures employed by Villard, but their studies strongly suggest that Villard did use some technique of manipulating squares to produce the main features of the tower and the lines of his drawing.

lard himself stated in his "Preface." That technique — the manipulation of geometrical forms — was what I have called the constructive geometry of mediaeval masons.

Because so many of the details of the *Sketchbook* apparently were intended to be explained orally, we have had to reconstitute this constructive geometry of Villard and Magister 2 by inference and deduction from the mere hints which their drawings and comments provide, and from comparison of the contents of the *Sketchbook* with other kinds of practical geometries compiled through the thirteenth century. These conditions make our case for the geometrical knowledge of master masons somewhat circumstantial for the period of High Gothic architecture. Fortunately, we enter upon much solider ground when we come to Late Gothic, thanks to several little books written by German master masons in the late fifteenth and early sixteenth centuries.

The earliest of these booklets to be printed were the two by Matthias Roriczer, *Büchlein von der Fialen Gerechtigkeit* and *Geometria deutsch*, the first published in 1486, the second a year or two later.[42] For our present purposes the *Geometria deutsch* is particularly interesting, since here is a treatise specifically devoted to "geometry" by a known mediaeval master mason. The booklet, or better still, pamphlet — for it contains only twelve pages — consists of lettered figures and brief explanations of the solution of nine problems: how to construct a right angle, a pentagon, a heptagon, and an octagon; how to find the length of the circumference of a circle; how to find the center of a circle with only part of the circumference known; how to construct a square and a triangle which have the same areas; how to set out the moldings and the finials for a gable; how to set out the plan of a gable.

It is not clear at first reading just what was Roriczer's intention in compiling this "German Geometry," since it is a somewhat incongruous agglomeration of simple geometrical problems along with techniques of architectural design and construction. One is reminded of the heterogeneous character of Villard's *Sketchbook* and tempted to comment that, whatever may have been the relationships between Gothic architecture and scholasticism, when mediaeval master masons did write books, they certainly did not reveal that penchant for systematic literary organization characteristic of scholastic treatises. This is not really surprising when one places these little books within the context of an oral rather than a written tradition. The Schoolmen taught from books, and in turn their own scholastic treatises were shaped by the techniques which they developed for teaching from books. Mediaeval masons did not teach from books,

[42] Although neither of these booklets has received a critical edition, both have been reprinted several times: the *Büchlein von der Fialen Gerechtigkeit* by Karl Heideloff (Nürnberg, 1844), A. Reichensperger (Trier, 1845), Karl Schottenloher (Regensburg, 1923), and Ferdinand Geldner (Wiesbaden, 1965); the *Geometria deutsch* by Heideloff (Nürnberg, 1844), Sigmund Günther, "Zur Geschichte der deutschen Mathematik im fünfzehnten Jahrhundert," *Zeitschrift für Mathematik und Physik, Historisch-literarische Abtheilung*, xx (1875), 1–14; and Geldner (Wiesbaden, 1965). It was Geldner who established (pp. 70–71) that Roriczer wrote and published both works, contrary to the prevailing views that the author of the *Geometria deutsch* was either unknown or that he was a certain Hans Hösch.

but rather from memory and from experience in the techniques of the craft. When they did decide to describe some of these techniques in written words and illustrations, they had no established literary forms to follow. Consequently, it appears that they too wrote as they taught — by piling up one description after another of the particular rules and procedures of the craft, with little of that scholastic concern for placing these particulars into some sort of systematic framework.[43]

But if most masons acquired their technical knowledge and skills through oral transmission of the craft traditions, at least some master masons must have had access to knowledge not carried by those traditions. Whether this knowledge came directly from a literary source, or from someone else who had access to that source, the knowledge itself could easily and quickly lose its literary moorings and become a part of the oral traditions of the craft, if it proved useful in design and construction.

Roriczer's two little books provide a fascinating insight into the masons' thought processes in this respect. In his *Booklet on the Correct Design of Pinnacles*, which is entirely concerned with architectural design problems, he cites as the source for his design technique the Junkers of Prague, that is, the famous fourteenth-century Parler family of master masons from Prague.[44] Since there is no reference to a previous book on the technique which Roriczer himself is describing, one presumes that he had gotten his knowledge of the technique by the usual oral transmission of craft traditions. On the other hand, for his *Geometria deutsch* Roriczer may have used a fifteenth-century treatise on geometry, *De inquisicione capacitatis figurarum*, which he does not cite, although there are some striking parallels between passages in it and some of his own geometrical exercises. It is uncertain who was the author of *De inquisicione*, but the manuscript in which the treatise survives was compiled by a certain Magister Reinhard de Vurm before the middle of the fifteenth century. In 1457 it was in the possession of Johannes Fleckel, who carried it with him to Vienna when he entered the Dominican Order. At Fleckel's death the Dominican house sold the manuscript to Burchard Keck, a citizen of Salzburg, and upon Keck's death it passed to the Bibliotheca Regiae in Munich.[45] Since Roriczer spent his career as a master mason in southern Germany, with positions at Eichstätt, Nürnberg, Munich, and Regensburg, he

[43] While I continue to be intrigued by the central thesis of Professor Panofsky's *Gothic Architecture and Scholasticism* (Cleveland, 1957), it does seem that he was carried away by his argument in his discussion (p. 87) of the "scholastic disputation" between Villard de Honnecourt and Pierre de Corbie regarding the plan of an ideal chevet. Likewise, it seems to me, he has overdrawn (pp. 23–26) his portrait of the education and learning, not to speak of social and professional status, of the mediaeval architect. On the other hand, it might be profitable to rethink his thesis in terms of the argument of this present paper.

[44] The handiest guide to the enormous literature on the Parlers is Otto Kletzl's article on the family in Thieme-Becker, *Allgemeines Lexikon der Bildenden Künstler* (Leipzig, 1932), XXVI, 242–248.

[45] These facts about the history of the manuscript are known from inscriptions within the manuscript itself. See Maximilian Curtze's introduction to his edition of the text, " 'De Inquisicione Capacitatis Figurarum.' Anonyme Abhandlung aus dem fünfzehnten Jahrhundert," *Abhandlungen zur Geschichte der Mathematik*, Heft VIII (1898), 31–32.

might have had access to the *De inquisicione*, or at least to someone who knew the contents of the treatise. That someone may well have been the Bishop of Eichstätt, Wilhelm von Reichenau, to whom Roriczer dedicated his booklet on pinnacles. In the dedication Roriczer mentioned that the bishop was a lover and patron of the "free art of geometry," and that they had discussed the subject together many times.[46] Incidentally, it is just this kind of rapport between the ecclesiastical patron and his master mason that must have provided the input of a great deal of "clerical" learning into the craft traditions of the Middle Ages.

The procedures which Roriczer used to construct a heptagon and an octagon are precisely the same as those in the *De inquisicione*.[47] In both cases the procedure was strictly by constructive geometry, that is, the manipulation of compass and straightedge to inscribe a heptagon within a circle and an octagon within a square. In neither case is the construction mathematically demonstrated to be correct.

On the other hand, Roriczer's method of determining the length of the circumference of a circle differed in important ways from that given in the *De inquisicione*. The solution to the problem had long been available in the Archimedean treatise, *On the Measurement of the Circle*, which in the twelfth century Gerard of Cremona had translated into Latin from an Arabic version. Throughout the rest of the Middle Ages European scholars copied, commented on, expanded, contracted, and otherwise transformed the *De mensura circuli*, with its three theorems concerning the area and circumference of the circle.[48] It is the third theorem which concerns us here: "Every circumference of a circle exceeds three times its diameter by an amount less than one seventh and more than 10 parts of 71 parts of the diameter."[49] Here is Roriczer's version of Archimedes' theorem:

Whoever wishes to make a circular line straight, so that the straight line and the circular

[46] Matthäus Roriczer, *Das Büchlein von der Fialen Gerechtigkeit. Faksimile der Originalausgabe Regensburg 1486* und Matthäus Roriczer, *Die Geometria deutsch: Faksimile der Originalausgabe Regensburg um 1487/88*, ed. Ferdinand Geldner (Wiesbaden, 1965), pp. [14], 45–46. Bishop Wilhelm was also a noted patron of architecture and was engaged in building projects in Eichstätt, Regensburg, Ulm, and Ingolstadt. See Geldner's comments on him in *ibid.*, p. 63.

[47] Roriczer, *Geometria deutsch*, pp. [34–35], 57–58; Curtze, "De Inquisicione," pp. 55–56. Roriczer's relatively complex procedure for constructing a pentagon (pp. [32–33], 56–57) is not contained in the *De inquisicione*, nor have I been able to find a literary source or parallel to it in the *practicae geometriae* already cited or in those compiled after 1300. For the latter I have used the following editions: *Geometria Culmensis. Ein agronomischer Tractat aus der Zeit des Hochmeister Conrad von Jungingen (1393–1407)*, ed. Hans Mendthal (Leipzig, 1886); "Die 'Practica Geometriae' des Leonardo Mainardi aus Cremona," in Part II of *Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance*, ed. Maximilian Curtze, *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, XIII (1902), 342–434; H. L. L. Busard, ed., "The Practica Geometriae of Dominicus de Clavasio," *Archive for History of Exact Sciences*, II (1962–66), 524–575; and Maximilian Curtze, ed., "Der Tractatus Quadrantis des Robertus Anglicus in Deutscher Übersetzung aus dem Jahre 1477," *Abhandlungen zur Geschichte der Mathematik*, IX (1899), 45–63.

[48] In his monumental work, *Archimedes in the Middle Ages*, Vol. I, *The Arabo-Latin Tradition* (Madison, 1964), Marshall Clagett has edited, translated, and commented on the numerous manuscripts of this mediaeval tradition.

[49] Clagett's translation, *ibid.*, p. 49, of Gerard's Latin translation as edited on p. 48.

are the same length: Make three circles next to one another and divide [the diameter of] the first circle in seven equal parts, with the letters as shown h : a : b : c : d : e : f : g. As far as it is from h to a, set a point behind it and there put an i. Thereby, as far as it is from i to k, equally as long is the circular line of one of the three circles which stand next to each other as shown in the attached figure.[50] (Fig. 3)

One could hardly ask for a more revealing example of the constructive geometry of mediaeval masons. Finding the circumference of a circle (for columns, pillars, round towers, etc.) must have been a relatively common requirement of mediaeval building. By following the steps prescribed by Roriczer, any mason could, with only his compass and straightedge, "construct" a solution to the problem, without knowing either the Archimedean theorem or the proofs pertaining to this theorem. The *Geometria deutsch* thus clearly reveals how mediaeval masons approached geometrical problems which would seem to require some mathematical calculations, yet they managed to avoid those calculations through step-by-step manipulations of their working tools. This point can be emphasized by comparison of Roriczer's solution with that given in the *De inquisicione:*

Given the diameter, to find the circumference of a circle: Let it be that the circle is *a b* and the diameter *a b* of the circle is given as 14. Triple the diameter and it becomes 42. If you add to the product 1/7 of the said diameter, that is to say 2, there will be produced [the number] 44, which is the circumference of the circle. This is made clear by [theorem] 7 of the geometry of the three brothers.[51]

Immediately one recognizes that the author of *De inquisicione* has rendered the Archimedean theorem in the form of an arithmetical calculation, whereas Roriczer has presented it as a geometrical construction. Indeed, the language of Roriczer's formula suggests that he was hardly thinking of this as a mathematical problem. He seems to have been visualizing certain geometrical forms — a circle and a straight line — and asking himself, "How do you make a straight line that is as long as a circle is round?" This mental "set," it seems to me, differs importantly from the geometer's question, "Given the diameter, how do you find the circumference of a circle?" Secondly, one notices the merely prescriptive character of Roriczer's formula: if you want to solve *that* problem, then do it *this* way. He feels no compulsion to demonstrate the correctness of his prescription. On the other hand, the author of *De inquisicione* refers the reader to the proof of the theorem in the *Geometria trium fratrum.*[52] While this is rudimentary mathematical

[50] Roriczer, *Geometria deutsch*, p. [36]: "Wer ain gerunden riss scheitgerecht mach*en* wil das der scheitgerecht ris vn*d* das gerund ain leng sey So mach drew gerunde neben ain and*er* vn*d* tail das erst ru*n*d in sib*en* gleiche tail mit den puchstaben verzaichnet .h: .a: :b .c. :d. :e· :f· .g: Darnach als weit vo*m* .h· in das :a. ist da sez hind*er*sich ain pu*n*ckt da sez ain :i: Darnach als weit von dem :i pis zv dem :k: ist Gleich so lanck ist d*er* runden riss ainer in seiner rund*en*s d*er* drey neb*en* ain ander sten des ain figur hernach gemacht stet."

[51] Curtze, "De Inquisicione," p. 37: "*Datae dyametri circumferenciam circuli invenire.* Esto, ut sit circulus *a b*, et dyametrus eius data, verbi gracia 14, sit *a b*. Quam dyametrum tripla, et proveniunt 42. Producto si 1/7 dyametri praedictae, scilicet 2, addideris, 44, quae sunt circuli circumferencia, producuntur. Patet per 7^{am} geometriae trium fratrum."

[52] This latter work, known variously in the Middle Ages as the *Librum trium fratrum* or the *Verba filiorum Moysi filii Sekir,* was composed by three ninth-century Arabic mathematicians who were brothers. Translated by Gerard of Cremona, it provided further treatment of Archimedes' theorems

reasoning, it at least shows the author's recognition that geometrical theorems do have to be demonstrated.[53]

Another example of Roriczer's non-mathematical approach may be seen in the following formula for what is an acceptable sort of Euclidean problem:

Whoever wishes to make a square and a triangle so that the square and the triangle each contain as much as the other: then make an [equilateral] triangle, that is, a : b : c. Divide c b in three equal parts, that is, [set points at] d and e. Then make a square out of [the line] c e [which] will become f g. Thereby the square contains equally as much as the triangle, as the example herewith shows.[54] (Fig. 4)

But the illustration merely shows the configuration of the two geometrical forms. Roriczer has certainly not proved that the two forms contain the same areas; he has only asserted that they do. In fact, the areas are only approximately the same — the area of the square is 4.000, while that of the triangle is 3.6742.

Again it is instructive to compare Roriczer's formula with the related one in *De inquisicione*:

Let there be triangle *a b c*. [Fig. 5] I make line *d e* equal to and equal distance from *b e* and I join *b d* and *e c*. According to Euclid I, 41, the parallelogram *b c e d* is double the triangle *a b c*; therefore, half of parallelogram *b c g f* is equal to triangle *a b c*. We must now seek the "quadratic side" of this parallelogram *b c g f*.[55] To side *b c* I add, in a continuous and straight line, *c h* equal to *g e*; having made the diameter *b h*, from its center I draw the semicircle *b k h*, and I produce *c k*. I say, therefore, that line *c k* is the "quadratic side" of parallelogram *b c g f*, and by consequence, of triangle *a b c*.[56]

It will be noted that the author of *De inquisicione* has also "constructed" a solution to the problem, but in so doing, he has relied on two Euclidean propositions. The first half of his solution depends on Proposition 41 of Book I of the *Elements*, which he dutifully cites. But the second half, in which he finds the square equal in area to the parallelogram, is based on Book II, Proposition 14, which for some

concerning the circle, sphere, and cone, as well as other geometrical and arithmetical problems. Gerard's translation has been edited and translated into English in Clagett, *Archimedes*, I, 238–355.

[53] For most theorems in the *De inquisicione* the author made a stronger effort to do so on his own. The demonstration of the Archimedean theorem for the circumference of the circle is rather sophisticated, and it is not really surprising that the author shied away from it. For the proofs of Archimedes and of the Three Brothers, see Clagett, *Archimedes*, I, 48–55, 264–279. On the other hand, a brief *Tractatus de quadratura circuli* — traditionally attributed to Campanus de Novara, but authorship and date uncertain — contains a solution of the problem by means of "constructive geometry" that is similar to the one used by Roriczer. For the text, see *ibid.*, pp. 590–93.

[54] Roriczer, *Geometria deutsch*, p. [38]: "Wer machen wil ain firung vn*d* ain driangel das die firung vn*d* der driangel yedlichs als vil in im helt als das ande*r* So mach ain driangel das ist :a: :b: :c: tail vom .c. pis zv dem :b: in drew gleiche tail das ist .d. .e. Darnach mach ain firung aus dem :c: :e: wirt .f. .g. So helt dy firung gleich als vil in als d*er* driangel des ain exempel he*r*nach gemacht stet."

[55] That is, the side of a square equal in area to the parallelogram *b c g f*.

[56] Curtze, "De Inquisicione," pp. 56–57: "Sit trigonus *a b c*. Ducam lineam *d e* aequalem et aeque distantem *b e* et coniungam *b d* et *e c*; ergo per quadragesimam primam primi EUCLIDIS parallelogrammum *b c e d* est duplum ad trigonum *a b c*, ergo medietas parallelogrammi *b c g f* est aequalis trigono *a b c*. Huius ergo parallelogrammi *b c g f* quaeratur latus tetragonicum sic. Lateri *b c* adiungam in continuum et directum lineam *c h* aequalem *g e*, et facta dyametro *b h* et centro in medio eius circinabo semicirculum *b k h* et producam *c k*: dico igitur, quod linea *c k* est latus tetragonicum parallelogrammi *b c g f*, et per consequens trianguli *a b c*."

reason he does not cite; consequently, his demonstration is left hanging a bit. Nevertheless, there are important differences between his and Roriczer's constructions. In the first place, the procedure in *De inquisicione* produces a geometrically correct solution, and not just an approximate one like Roriczer's. Secondly, this procedure applies to *any* triangle, and not just the equilateral triangle of Roriczer's formula. Finally, the author of *De inquisicione* assumes that the reader will understand his solution by reference to one of the Euclidean propositions on which the solution is based. Roriczer makes no such assumption about the mathematical interests or abilities of the reader; he simply says to do it this way. Whether Roriczer himself would have understood the Euclidean propositions is a moot question; certainly he did not require that understanding of his readers.

A final point of comparison between the *Geometria deutsch* and the *De inquisicione*: if Roriczer did have access to the latter, or to someone who knew its contents, he borrowed from the treatise only those formulae which could be entirely expressed in terms of constructive geometry, and he avoided all those others which required some kind of Euclidean mathematical reasoning. This point brings out an interesting contrast between the geometry of mediaeval masons and classical Greek geometry as developed by Euclid in the *Elements*. Both began with the same presupposition, namely, that all geometrical constructions must be possible with the use of a few simple tools or instruments. But Euclid accepted that presupposition as a theoretical restriction in order to prescribe the limits within which he would rigorously develop his arguments. Mediaeval masons, on the contrary, made that presupposition as a practical necessity because they lacked the ability in mathematical reasoning to push beyond the mere manipulation of their tools into the Euclidean world of definitions, postulates, axioms, propositions, and proofs. For Euclid the construction of a geometrical figure with compass and straightedge was merely a part — and not an absolutely necessary part — of his mathematical exercise; there remained the more difficult and important task of demonstrating the mathematical correctness of the construction.[57] For Roriczer and his fellow masons, such a construction was, geometrically or mathematically speaking, the end of the exercise; the next task was not to prove its mathematical correctness, but to transform the geometrical construct into an architectural form in stone.

Some of the ways in which mediaeval masons transformed geometrical figures into architectural forms were described and illustrated in three other small treatises by late mediaeval German master masons. The best known of these is Roriczer's *Büchlein von der Fialen Gerechtigkeit*. In his dedication of the booklet to Bishop Wilhelm von Reichenau, Roriczer explained that since every art has its

[57] On the formal structure of Euclidean propositions and the place of geometrical constructions within these, see Thomas Heath, *A History of Greek Mathematics* (Oxford, 1921), I, 370–371. On the Euclidean techniques of construction with the straightedge and the "collapsing compass," see Julian L. Coolidge, *A History of Geometrical Methods* (Oxford, 1940), pp. 43–45; and Howard W. Eves, *A Survey of Geometry* (Boston, 1963), I, 183–184.

own materials, forms, and measures (shades of Dominicus Gundissalinus!), he wished to set forth the basic principles of these for the art of masonry, as founded on the art of geometry. "I have [tried], with the help of God, to make clear this previously mentioned art of geometry and to explain, in the first place, the beginning of drawn-out stonework — how and in what measure it arises out of the fundamental basis of geometry through the manipulation of compasses and [how it] should be brought into the correct proportions."[58]

The booklet is in fact entirely devoted to the technique of setting out the groundplan and elevation of a pinnacle, but the procedure of moving step by step is the same as that in the *Geometria deutsch*. Roriczer begins, "If you wish to draw the groundplan for a pinnacle, according to the stonemason's art and with the correct geometry, then begin by drawing a square, as it is shown here with the letters a, b, c, d."[59] Inside this square Roriczer inscribed a second square at a forty-five degree angle to the first, and inside the second he inscribed a third square in the same manner. (Fig. 6) He then rotated the second to make the sides of all three squares parallel. Having obtained the basic outline of the groundplan of the pinnacle by this manipulation of three squares, Roriczer proceeded to determine the details of both the plan and the elevation in a step-by-step elaboration of this manipulative technique. The entire process required 234 separate steps, which he illustrated with eighteen figures, the last three showing the completed geometrical design for the groundplan and elevations. (Fig. 7) Roriczer finished his instructions with these comments: "Thereupon, place the cap of the pinnacle on the body of the pinnacle and erase all drawing lines, so that there remain only correct lines which are necessary for the pinnacle. Accordingly, the figure is called a correct pinnacle, drawn from the groundplan."[60] In short, this is the famous technique of quadrature, whereby the elevation is derived from the square, the basic geometrical figure of the groundplan. But it cannot be too strongly emphasized that the entire operation consisted simply of the manipulation of geometrical figures through a long series of carefully prescribed steps, or that it was quite devoid of mathematical formulae and calculations. What Roriczer did in his little book was to provide in writing the detailed exposition of a particular design problem and its solution.[61] The literary form of this exposition

[58] Roriczer, *Büchlein*, p. [14]: "Hab ich mit der hilff gotes ettwas berurter kunst d*er* geometrey zuerleutern Un*d* am erst*en* dasmale den anefang des aussgezogens stainwerchs wie vn*d* in welcher mass das auss dem grunde d*er* geometrey mit austailung des zirckels herfurkomen vnd in die rechten masse gebracht werden solle Zuerclern. . . ." Cf. John Papworth's translation of Roriczer's dedication in Elizabeth Holt, ed., *A Documentary History of Art*, Vol. I, *The Middle Ages and the Renaissance* (Garden City, N. Y., 1957), pp. 96–97.

[59] Roriczer, *Büchlein*, p. [15]: "Wilt dv ain grvndt reyss*en* zw ainer vialen: nach stainmezischer art: avss der rechten geometrey So heb an vn*d* mach ain virvng als hernach bezaichnet ist mit den puchstab*en* :a:b:c:d·"

[60] *Ibid.*, p. [29]: "Darnach so sez den risen der fialen auf den leib der fialen vnd du al tail riss noher so pleibt nur dy rechten riss dy noturftig sein in d*er* fialen Darnach so haist dj figur ain rechte viale*n* aus gezoge*n* auss dem grunt."

[61] This is not the place to pursue the question of why Roriczer and other contemporary master masons felt inclined to publish these descriptions of the technique of quadrature. I hope later to provide a more intensive study of the problem of the "secret" of the mediaeval masons.

suggests that it is nothing more than a written record of the kind of oral teaching that was traditional in the craft. It is perhaps not unlike the exposition which a master mason might have given his apprentice in explaining some of the sketches and curt comments of Villard and Magister 2.

The same design problem and pretty much the same technique for resolving it were set forth in another *Fialenbüchlein* by Roriczer's contemporary, Hans Schmuttermayer.[62] In a prefatory statement Schmuttermayer provides a definition of geometry and its use in building construction that is very close to the point of view expressed in the "Articles and Points of Masonry" and to the content of constructive geometry as expounded in this present paper. Schmuttermayer explains that he is writing the book

> . . . for the instruction of our fellowmen and all masters and journeymen who use this high and free art of geometry, in order that their feelings, speculations, and imaginings can, with thought, be better subjected to the correct rules of measured stonework and take root. Fundamentally, this art is freely and truly planted and founded on the center point of the circle, together with its circumference of correctly set point and construction. [I explain these matters,] not for my own reputation, but more to praise the fame and reputation of our forerunners, the inventors of this high art of building construction, which from the beginning has had its true base in the level, set-square, triangle, compass, and straightedge, but which is now pursued with greater subtlety, higher understanding, and deeper reckoning. Thus have I, Hans Schmuttermayer of Nürnberg, correctly shown the art of such measured stonework in the square and round [parts] of the pinnacle, the gable, and the pillar, with all of the things belonging to these, according to the new as well as the old art. . . . I have not discovered these things by myself, but have learned them from many other great and famous masters, such as the Junkers of Prague, and Masters Ruger and Nicholas of Strasbourg, who for the most part brought this new art to light, along with many others.[63]

Schmuttermayer's technique for designing a pinnacle was basically the same as that used by Roriczer, but he clarified the procedure somewhat by separating the inscribed squares, placing them in a row, and giving them individual letter-symbols for easy reference. (Fig. 8) Then in the customary manner of carefully prescribed procedures, he transferred the length of the sides of these various squares to establish the dimensions and outlines of the essential features of the pinnacle.

[62] The little pamphlet was edited by A. Essenwein, "Hans Schmuttermayer's Fialenbüchlein,' *Anzeiger für Kunde der Deutschen Vorzeit*, XXVIII (1881), 73–78.

[63] *Ibid.*, p. 73: ". . . vntterweysung vnnserm nachsten vñ allē maisteren vñ gesellen die sich diser hohen vñ freyen kunst der Geometria geprauchen ir gemute speculirung vnd ymaginacion dem warē grunt des maswercks pass zuuntterwerffen nach gedencken vnd ein zu wurtzeln. Auch fundamentlicher die art so auss dem Centrum des zirckels mitsamt seines vmbschweiffs warer saczung punct vñ austeylung dest freyer vñ warhafftiger eingepflanczt vnd gegrundt werden. Vnd nit vmb meiner eygen Ere willen. Sunder mer zupreyse rum vnd lob der altten vnnser vorgeer seczer vñ vinder diser hohē kunst des pauwercks die auss der wage. winckelmoss. triangel. zirckel. vñ linial. vrsprunglichē iren warē grunt habē. vñ nu mit der scherff. subtilitet. hoher synne. vñ tieffer rechnūg. yecz ersucht ist. Hyrumb hab ich Hanns schmuttermayer von Nurmberg die art solichs maswercks. virung. rotund. der violn. winperg. vñ der pfeyler mit aller irer zugehorungē auff die new mitsamt der alttē art gerecht gemacht. . . . Vnd hab solichs auss mir selber nit erfunden. sunder von vil andern grossen berumbtē maisteren. Als die Junckhern von prage. Maister ruger. Niclas von straspurgk. Der dan am mainsten die new art an das licht gepracht mitsamt vil andern genomen."

The result was a "geometrical construct" of the pinnacle which he then transformed into an architectural drawing showing the details of moldings, crockets, and finial. (Fig. 9) In these same figures Schmuttermayer illustrated the application of this technique to the design of a gable or canopy. His illustrations, which are not nearly so well known as those of Roriczer, reveal more explicitly the use of "geometry" by mediaeval master masons in manipulating geometrical forms to produce the framework in which they could then develop their own individual architectural forms. Comparison of Schmuttermayer's gable with the one produced by Roriczer in the *Geometria deutsch* (Fig. 10) shows that while their technique of designing by constructive geometry was basically the same, the end results — in both overall composition and formal details — were distinctively different.

The development of variations on a theme constitutes a fundamental characteristic of the constructive geometry of mediaeval masons, and it is virtually stated as a matter of principle in another little book by a German master mason named Lorenz Lechler. Written in 1516 for the instruction of his son Moritz into the techniques and skills of the masons' craft, this small work bears no title and is generally referred to simply as Lechler's *Unterweisung*.[64] Because of the variety of design and construction problems with which Lechler concerned himself, his treatise is more interesting than the two booklets on pinnacles by Roriczer and Schmuttermayer. Lechler gave particular attention to the design of the templates used in cutting stones for window mullions, tracery, vault ribs, bases and capitals of pillars, and moldings for gables, pinnacles, and buttresses. Since I have published a detailed study of Lechler's prescriptions for designing templates by the technique of quadrature, it will be necessary here to review his system only very briefly.[65] It was based on a modular unit determined by the choir wall of the church: "Take the wall thickness of the choir, whether it be small or large, then draw two squares through one another; therein will you find all templates, just as you will find them drawn in this book."[66] As can be seen from Fig. 11, these two squares of the same size were inscribed over each other at a forty-five degree angle. One of these was divided into nine squares, and the centermost square then provided the frame for inscribing three small squares in the manner already made familiar by Roriczer and Schmuttermayer. This grid of inscribed squares determined the length and breadth of large and small mullions. (Fig. 11 bottom) Another combination of circles and squares within this same framework provided the dimensions of the cross-sections for large and small vault ribs. Finally, by manipulating compasses and straightedge on the corners of the original overlapping squares, Lechler obtained the geometrical forms of large and small vaulting shafts, along with their bases and capitals. Though by now it sounds like a refrain,

[64] The unique text of Lechler's book is contained within a late sixteenth-century MS copy (Cologne, Historisches Archiv, Handschrift Wf. 276*, fols. 41–56v); it was published in an unsatisfactory edition by August Reichensperger, *Vermischte Schriften über christliche Kunst* (Leipzig, 1856), pp. 133–155.

[65] Shelby, "Mediaeval Masons' Templates," pp. 147–153.

[66] Handschrift Wf. 276*, fol. 44 (Reichensperger's edition, p. 135): "Item so nimb die mauer dickhe, von dem khor, er sey̋ khlein oder gros so reiss Zwo fürung durch einander, darinen fintestu alle bredter, wie du dan alhie in desem Buech gerisen findest. . . ."

one must repeat that throughout Lechler's prescriptions there were no mathematical calculations involved, and the entire procedure was accomplished by manipulating geometrical figures. Even the numerical ratios of seven to five which keep cropping up in Lechler's prescriptions turn out to be inevitably determined by his geometrical technique of inscribing squares.[67]

One of the more significant points which emerges from a study of Lechler's technique for designing templates is that while it was certainly prescriptive, it was not rigidly restrictive. The present study helps to see why this was the case. The constructive geometry of mediaeval masons was prescriptive in that it consisted of carefully prescribed steps which the masons were taught to follow. But since they were scarcely concerned with mathematical preciseness or correctness, those steps could be altered at will. That is to say, there were no logical or mathematical rules which they were obligated to follow; they were restricted only by their own skill and inventiveness in manipulating geometrical forms with the tools at their disposal, and by their willingness, or unwillingness, to change the prescriptions which had been handed down to them through the craft traditions.

* * *

Mediaeval masons insisted that their whole craft was based on the "art and science of geometry." It has been the purpose of this paper to reconstruct the character and content of the geometrical knowledge of mediaeval master masons from the few literary remains of the masons themselves. As reconstructed from these writings, this geometry scarcely resembles either the classical geometry of Euclid and Archimedes, or the mediaeval treatises on *practica geometriae*. Mathematically speaking, it was simple in the extreme; once it is recognized that there was virtually no Euclidean-type reasoning involved, the way is cleared for understanding the kind of geometrical thinking which the masons did employ. This non-mathematical technique I have labeled constructive geometry, to indicate the masons' concern with the construction and manipulation of geometrical forms. It becomes evident that the "art of geometry" for mediaeval masons meant the ability to perceive design and building problems in terms of a few basic

[67] Lechler's use of geometrical constructions and simple arithmetical ratios calls to mind the mathematical calculations of the sixteenth-century Spanish master mason, Rodrigo Gil de Hontañon. See George Kubler, "A Late Gothic Computation of Rib Vault Thrusts," *Gazette des Beaux-Arts*, 6th Ser., xxvi (1944), 138–148. Rodrigo's arithmetical computations are considerably more sophisticated than anything to be found in Villard or the German master masons, but his geometrical formulae for determining the thickness of the buttresses for a barrel vault appear to be within the traditions of the constructive geometry of mediaeval masons. See his text and illustrations in the only modern, and unsatisfactory, edition by Ricardo de Mariátegui, "Compendio de Arquitectura," *Arte en España*, vii (1868), 174–176. Rodrigo was also knowledgeable in at least some aspects of Renaissance theory and practice of architecture, and this, combined with the textual problems of his writings being transmitted only through a seventeenth-century architectural treatise, raises several questions which have to be sorted out before one can make a confident statement on the extent to which his mathematical computations reflect the practice and knowledge of mediaeval masons. I have had the opportunity to study briefly Prof. John Hoag's very fine, but unpublished, dissertation, "Rodrigo Gil de Hontañon: His Work and Writings" (Yale, 1958). Since this study provides a firm basis for the resolution of some of these problems, it is to be hoped that Prof. Hoag will soon put into print the results of his research.

geometrical figures which could be manipulated through a series of carefully prescribed steps to produce the points, lines, and curves needed for the solution of the problems. Since these problems ranged across the entire spectrum of the work of the masons — stereotomy, statics, proportion, architectural design and drawing — the search by modern scholars for the geometrical canons of mediaeval architecture is appropriate enough, so long as we keep clearly in mind the kind of geometry that was actually used by the masons. The nature of that geometry suggests that these canons, when recovered, will not be universal laws which will at last provide *the key* to mediaeval architecture; rather, they will be particular procedures used by particular master masons at particular times and places.

SOUTHERN ILLINOIS UNIVERSITY